Advanced Linear Algebra - Algorithms

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1 Introduction

This resource is designed to help you master the *Additive and Multiplicative Principles* from Weeks 1 and 2 of Discrete Mathematics (MA284). The notes are styled as a *tutorial*, and promote learning through a process of self-directed examination and assessment. This is done through a set of exercises, which are based on WeBWorK, the system used for online assessment for this module. However, the exercises in this tutorial do not contribute to your continuous assessment score. Instead, they allow you to test your knowledge and skills in Discrete Mathematics. If you find you can do these exercises without difficulty, then you have validated your own learning. If no, then you will know where to focus the efforts.

You can attempt exercises as many times as you like. When you submit your answer, you will be informed it is correct or not. If not, you can re-attempt the question, with new, randomly generated data.

These notes do not contain all the core material on the Additive and Multiplicative Principles. Therefore, before proceeding, please review

- the the lecture notes from those classes, which are available from the MA284 website.
- Section 1.1 of Oscar Levin's Discrete Mathematics: an open introduction.

This tutorial was produced by Niall Madden as part of a project of the module CEL263: Learning Technologies. If you have any comments, please contact Niall.

Some of the exercises and examples here are taken, or adapted, from Discrete Mathematics: an open introduction, with attribution.

2 The Additive Principle

Definition 2.1 (Additive Principle)**.** The additive principle states that if event A can occur in m ways, and event B can occur in n *disjoint* ways, then the event "A or B" can occur in $m + n$ ways.

See DDOI, Section 1.1

The key to this definition is understanding what *disjoint* means. Simply put, two events are disjoint if they cannot both occur in the same context.

Example 2.2 (Disjoint events)**.** If you visit the animal rescue shelter with the purpose of adopting a pet then the the event of adopting a cat is disjoint from the event of adopting a dog.

Example 2.3 (Disjoint and non-disjoint events)**.** The events of choosing playing cards that belong to the heart or club suits are disjoint, since no card can below to both suits.

The events of choosing red card and a Number 4 card are not disjoint, since there are two red "4" cards.

Exercise 2.4 (When can we use the Additive Principle?)**.**

1. Can we use the additive principle to determine how many two letter "words" begin with either A *or* B? (Choose one: yes / no)
2. Can we use the additive principle to determine how many two letter "words" contain either A *or* B? (Choose one: yes / no)

Solution.

1. It is not possible for a word to begin with both A *and* B, so these events are disjoint, and the Additive Principle applies.
2. A word *can* contain both A *and* B, so these events are not disjoint, and the Additive Principle does not apply.

Exercise 2.5 (Using the Additive Principle)**.** Use the Additive Principle to solve these problems.

You visit the University Animal Shelter to adopt a new pet. The shelter has 8 dogs, 5 cats, and 3 ducks in need of a home.

1. How many choices do you have for a new pet?
2. How many choices do you have for a new four-legged pet?

Solution.

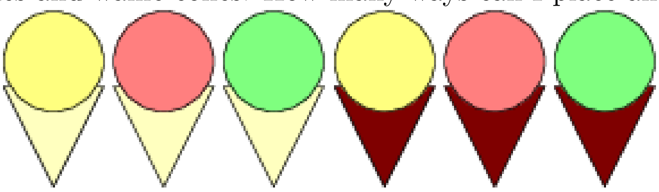
1. You have 16 choices for a pet.
2. You have 13 choices for a four-legged pet.

3 The Multiplicative Principle

Definition 3.1 (The Multiplicative Principle)**.** The multiplicative principle states that if event A can occur in m ways, and each possibility for A allows for exactly n ways for event B, then the event "A and B" can occur in $m \times n$ ways.

See DDOI, Section 1.1

Example 3.2 (Using the Multiplicative Principle)**.** Suppose we go to our favourite ice-cream shop where they stock three flavours: vanilla, strawberry and mint. They had two types of cone: plain cones and waffle cones. How many ways can I place an order (for 1 cone and 1 scoop?).



Exercise 3.3 (Using the Multiplicative Principle)**.** Use the Multiplicative Principle to solve these problems.

You return to the University Animal Shelter, which has has 5 cats, 8 dogs, and 3 ducks in need of a home.

1. You wish to adopt one cat and one dog. How many ways can you do this?

2. How many ways can you choose one cat, one dog, and one duck to adopt? $\boxed{}$

Solution.

1. You have $5 \times 8 = 40$ choices for a cat and a dog
2. You have 120 ways of choosing one of each animal.

4 Further exercises

Exercise 4.1.

Our Indiscrete Mathematics course has

- 25 students from the the College of Arts, 18 of whom are female;
- 25 students from the the College of Engineering and Informatics, 5 of whom are female;
- 39 students from the the College of Science, 19 of whom are female.

How many ways can we choose a single class rep?
Answer: ____________

How many ways can we choose **three reps**, one from each of the three Colleges?
Answer: ____________

How many ways can we choose **three reps**, one from each of the three Colleges, so that **exactly one** is female?
Answer: ____________

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The next two exercises involve sets. The second one is based on Exercises 1.1.5 and 1.1.6 from the textbook.

Exercise 4.2.

The sets A and B are such that $|A| = 10$, $|B| = 2$, and $|A \cap B| = 0$.

Then $|A \cup B|$ is ______, and $|A \setminus B|$ is ______.

The sets C and D are such that $|C| = 10$, $|D| = 3$, and $|C \cap D| = 1$.

Then $|C \cup D|$ is ______, and $|C \setminus D|$ is ______.

The sets E and F are such that $|E| = 18$, $|F| = 21$, and $|E \cup F| = 31$.

Then $|E \cap F|$ is ______, and $|E \setminus F|$ is ______.

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Exercise 4.3.

The sets A and B are such that $|A| = 19$, $|B| = 18$.

The largest possible value of $|A \cup B|$ is ______.

The smallest possible value of $|A \cup B|$ is ____.

The largest possible value of $|A \cap B|$ is ____.

The smallest possible value of $|A \cap B|$ is ____.

The value of $|A \cap B| + |A \cup B|$ is ____.

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Exercise 4.4.

How many "words" can you make from the letters DEE?
Answer: ______

How many "words" can you make from the letters NORE?
Answer: ______

How many "words" can you make from the letters CORRIB?
Answer: ______

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Exercise 4.5.

A Discrete Mathematics student comes from the village of KILLYIKYAKYODLE. As an hilarious jape, on the way home from a late-night study session, she decides to rearrange the letters on one of the sign-posts.

How many arrangements of the letters in KILLYIKYAKYODLE are possible?

Answer: ______

Of these arrangements, how many have all the L's together?
Answer: ______

Of these arrangements, how may have all the letters in alphabetical order?
Answer: ______

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