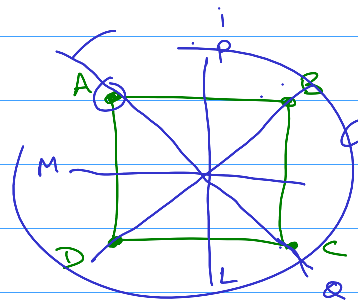
Recall A group G acts on a set S if every element of G permutes the elements of S, subject to

$$\boxed{\begin{array}{ll} id_G \cdot x = x & \text{for all } x \in S \\ g\cdot(h \cdot x) = (gh)\cdot x & \text{for all } x \in S \text{ and } g,h \in G \end{array}}$$

Example 1 The dihedral group D_{2n} acts on the vertex set of the regular n-gon.



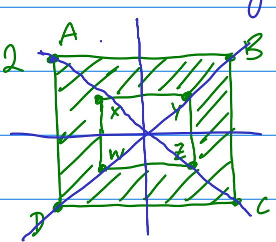
$G = D_8$

$S = \{A, B, C, D\}$

There is one orbit consisting of all four elements $\{A, B, C, D\}$

The stabilizer of A is $\{id, T_Q\}$ - elements of G fixing A

Example 2



The symmetry group of a square annulus is the same as that of a square : D_8

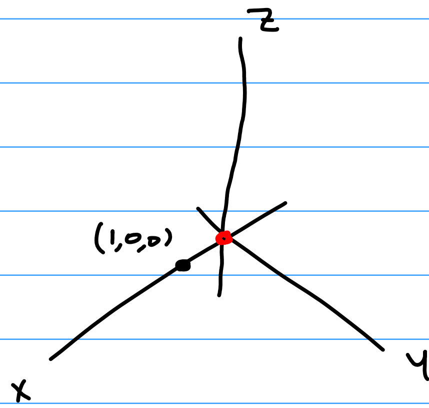
Vertex set $\{A, B, C, D, X, Y, Z, W\}$

Orbits $\{A, B, C, D\}$ and $\{X, Y, Z, W\}$

Action of $\boxed{GL(3,\mathbb{R})}$ on $\mathbb{R}^3$

How many orbits?

What is in the orbit of $\begin{pmatrix}1\\0\\0\end{pmatrix}$?



How many different vectors can we get by multiplying $\begin{pmatrix}1\\0\\0\end{pmatrix}$ on the left by an invertible matrix?

$$\begin{pmatrix} a & * & * \\ b & * & * \\ c & * & * \end{pmatrix}\begin{pmatrix}1\\0\\0\end{pmatrix}$$

Every vector that is the first column of an invertible 3×3 matrix is in the orbit of $\begin{pmatrix}1\\0\\0\end{pmatrix}$ - every nonzero vector.

Answer

Two orbits

One with $\begin{pmatrix}0\\0\\0\end{pmatrix}$

One with all other elements of $\mathbb{R}^3$