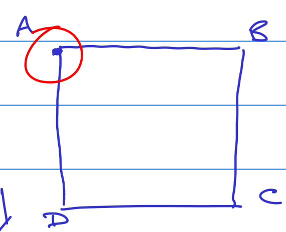
Group Actions   G acting on a set S

Group elements "do something"

They act as permutations – they move other things around in some set S.

$id_G \cdot x = x$ for all $x \in S$ $g \cdot (h \cdot x) = (gh) \cdot x$ for all $g, h \in G$, and $x \in S$.

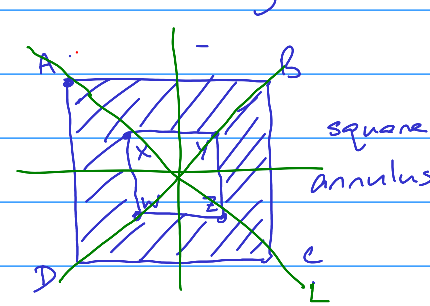
Example - Dihedral Groups



D_8 acts on the vertex set $\{A, B, C, D\}$ of the square.

If we look at what happens to A under the action of D_8, there are symmetries that take $A \to A$, $A \to B$, $A \to C$ and $A \to D$

We say the orbit of A is $\{A, B, C, D\}$. This action has a single orbit



Square annulus has vertex set $\{A, B, C, D, X, Y, Z, W\}$

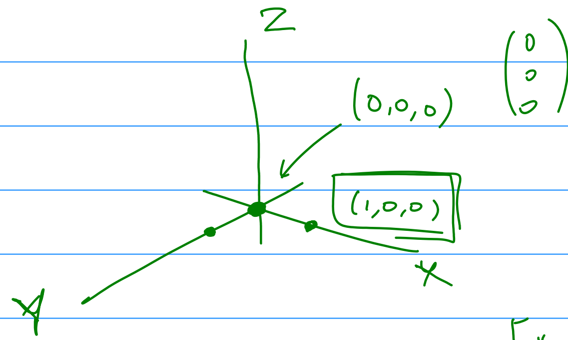
Symmetry group is D_8

Orbits are $\{A, B, C, D\}$ and $\{X, Y, Z, W\}$

Stabilizer of $A = \{id, T_L\}$ ←

$GL(3,\mathbb{R})$ acting on $\mathbb{R}^3$ as linear transformations.

How many orbits?



What can we reach by

$$\begin{pmatrix} a_{11} & * & * \\ a_{21} & A & \\ a_{31} & & \end{pmatrix}\begin{pmatrix}1\\0\\0\end{pmatrix} = \begin{pmatrix}a_{11}\\a_{21}\\a_{31}\end{pmatrix}$$

$\uparrow$ any element of $GL(3,\mathbb{R})$

first column of A?

$$\begin{bmatrix} * & & \\ * & & \\ * & & \end{bmatrix}$$

The orbit of $\begin{pmatrix}1\\0\\0\end{pmatrix}$ consists of all vectors that can be the first column of an element of $GL(3,\mathbb{R})$ - all non-zero vectors.

Two orbits: one with $\begin{pmatrix}0\\0\\0\end{pmatrix}$ and one with every other element of $\mathbb{R}^3$.