

Challenge for Week 8

$$\text{Let } G = \left\{ \begin{pmatrix} a & -b \\ b & a \end{pmatrix} : a, b \in \mathbb{R}, a^2 + b^2 = 1 \right\}.$$

- ▶ Show that G is a group under matrix multiplication.
- ▶ G acts on \mathbb{R}^2 as a group of invertible linear transformations (or by matrix-vector multiplication if we interpret \mathbb{R}^2 as the set of column vectors of length 2). What are these transformations, geometrically? Give a geometric description of the orbits of this action.