Group Actions

Here's that permutation in S_{14} again, and its disjoint cycle representation.

$$\pi = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 & 10 & 11 & 12 & 13 & 14 \\ 11 & 9 & 8 & 2 & 5 & 1 & 12 & 14 & 6 & 7 & 3 & 13 & 10 & 4 \end{pmatrix}$$

$$\pi = (1\ \ 11\ \ 3\ \ 8\ \ 14\ \ 4\ \ 2\ \ 9\ \ 6)(7\ \ 12\ \ 13\ \ 10)(5).$$

There are cycles of length 9, 4 and 1.
Note that π has order $36 = \mathrm{lcm}(9, 4)$, as an element of S_{14}.
So π generates a cyclic subgroup of S_{14}, of order 36.
Let's call this subgroup G.

Action and Orbits

In S_{14}, $\pi = (1\ \ 11\ \ 3\ \ 8\ \ 14\ \ 4\ \ 2\ \ 9\ \ 6)(7\ \ 12\ \ 13\ \ 10)(5)$.

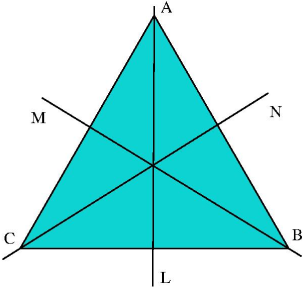
We have already referred to the sets $\{5\}$, $\{7, 10, 12, 13\}$, and $\{1, 2, 3, 4, 6, 8, 9, 11, 14\}$ as the orbits of the permutation π.

From the viewpoint of group actions, we say that the group $G = \langle\pi\rangle$ acts on the set $\{1, 2, \ldots, 14\}$. This means that every element of G permutes the elements of $\{1, 2, \ldots, 14\}$.

We say that the sets $\{5\}$, $\{7, 10, 12, 13\}$, and $\{1, 2, 3, 4, 6, 8, 9, 11, 14\}$ are the orbits of this action.
This means that given elements x and y of $\{1, 2, \ldots, 14\}$, there is an element σ of G that takes x to y if and only if x and y are in the same orbit.

Note: The full symmetric group S_{14} also acts on $\{1, 2, \ldots, 14\}$.
For this action there is only one orbit, the full set $\{1, 2, \ldots, 14\}$.

Another example: dihedral groups



D_6 acts on the set $\{A, B, C\}$ of vertices of the triangle.

- $R_{120} \cdot A = C$
- $T_L \cdot A = A$
- $T_M \cdot C = A$

There is a single orbit consisting of all three vertices.
The elements of D_6 that send the vertex A to itself are the identity and the reflection T_L. We say that the stabilizer of A is $\{\mathrm{id}, T_L\}$. Note that this is a subgroup of D_6.

Note also that $2 \times 3 = 6$, where 2 is the order of the stabilizer of A, 3 is the number of elements in the orbit of A, and 6 is the order of the group D_6.