Challenge for Week 4

The Euclidean plane $\mathbb{R}^2$ is a group under vector addition. The elements are ordered pairs (a, b) (points in the plane) and the addition is defined by $(a, b) + (c, d) = (a + c, b + d)$. The group $\mathbb{R}^2$ is also a real vector space, which means that its elements can be multiplied by real numbers as well as added together.

What are the nontrivial proper subgroups of $\mathbb{R}^2$ that are also closed under multiplication by real numbers?
If H is such a subgroup of $\mathbb{R}^2$, what do the left cosets of H determined by different elements of $\mathbb{R}^2$ look like?

Hint/Remark: There are very many nontrivial proper subgroups of $\mathbb{R}^2$ of this type, but geometrically they all look alike, and their left cosets have a nice geometric description too. A good answer to this challenge would be a geometric explanation with a picture.