

Week3, Challenge1

Let G be a group under binary operation \star with identity element e_G and H is a subgroup of G with identity element e_H .

Since e_H is an element of both H and G , we have

$$\begin{aligned} e_H &= e_H \star e_G && (x \star e_G = x \quad \forall x \in G) \\ &= e_H \star (e_H \star e_H^{-1}) && (x \star x^{-1} = e_G \quad \forall x \in G) \\ &= (e_H \star e_H) \star e_H^{-1} && (\star \text{ is an associative operation}) \\ &= e_H \star e_H^{-1} && (x \star e_H = x \quad \forall x \in H) \\ &= e_G && (x \star x^{-1} = e_G \quad \forall x \in G) \end{aligned}$$

Therefore, the identity of a subgroup is the identity of the group.