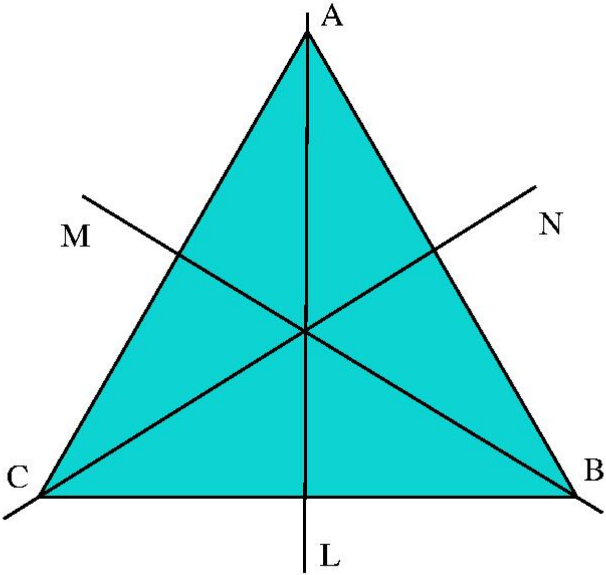
Lecture 3: Group of permutations and symmetries

A permutation of a finite set T is a *bijection* from T to T. This means that every element has a different image, and the image of the function is the whole set T. The permutations of a set form a group under composition.

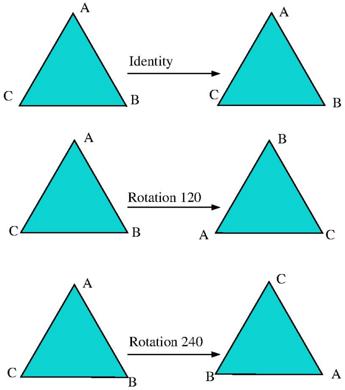
The group of all permutations of a set of n elements is called the symmetric group of degree n and denoted S_n.

How the composition operation works: the example of S_3.

The group of symmetries of the equilateral triangle



Symmetries of the triangle

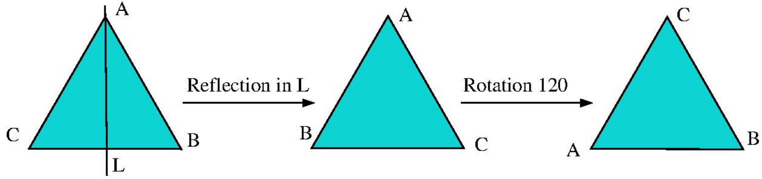


The Group Operation in D_6

The group D_6 of symmetries of the triangle has six elements.

$$D_6 = \{\mathrm{id}, R_{120}, R_{240}, T_L, T_M, T_N\}.$$

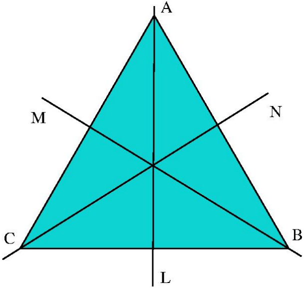
The group operation is composition, denoted by the symbol $\circ$. $R_{120} \circ T_L$ means "R_{120} *after* T_L, the symmetry obtained by applying T_L first and then R_{120}. We can figure out which one it is by watching what happens to the vertices in this composition of symmetries.



Comparing the final position to the starting position, we see that

$$R_{120} \circ T_L = T_M.$$

Group table for D_6



$\circ$	id	R_{120}	R_{240}	T_L	T_M	T_N
id	id	R_{120}	R_{240}	T_L	T_M	T_N
R_{120}	R_{120}	R_{240}	id	T_M	T_N	T_L
R_{240}	R_{240}	id	R_{120}	T_N	T_L	T_M
T_L	T_L	T_N	T_M	id	R_{240}	R_{120}
T_M	T_M	T_L	T_N	R_{120}	id	R_{240}
T_N	T_N	T_M	T_L	R_{240}	R_{120}	id

In general, the group of symmetries of the regular n-gon is denoted D_{2n} and called the dihedral group of order $2n$. It has $2n$ elements, n rotations (including the identity) and n reflections.

Weekly Challenge 2

Like a polygon, a 3-dimensional object has a group of symmetries, which includes rotations and reflections. This week's challenge is to give a description of the rotational symmtries of the cube. How many are there? What are the axes about which rotational symmetries occur, and what the the angles of rotation?