Turning A Tetrahedron

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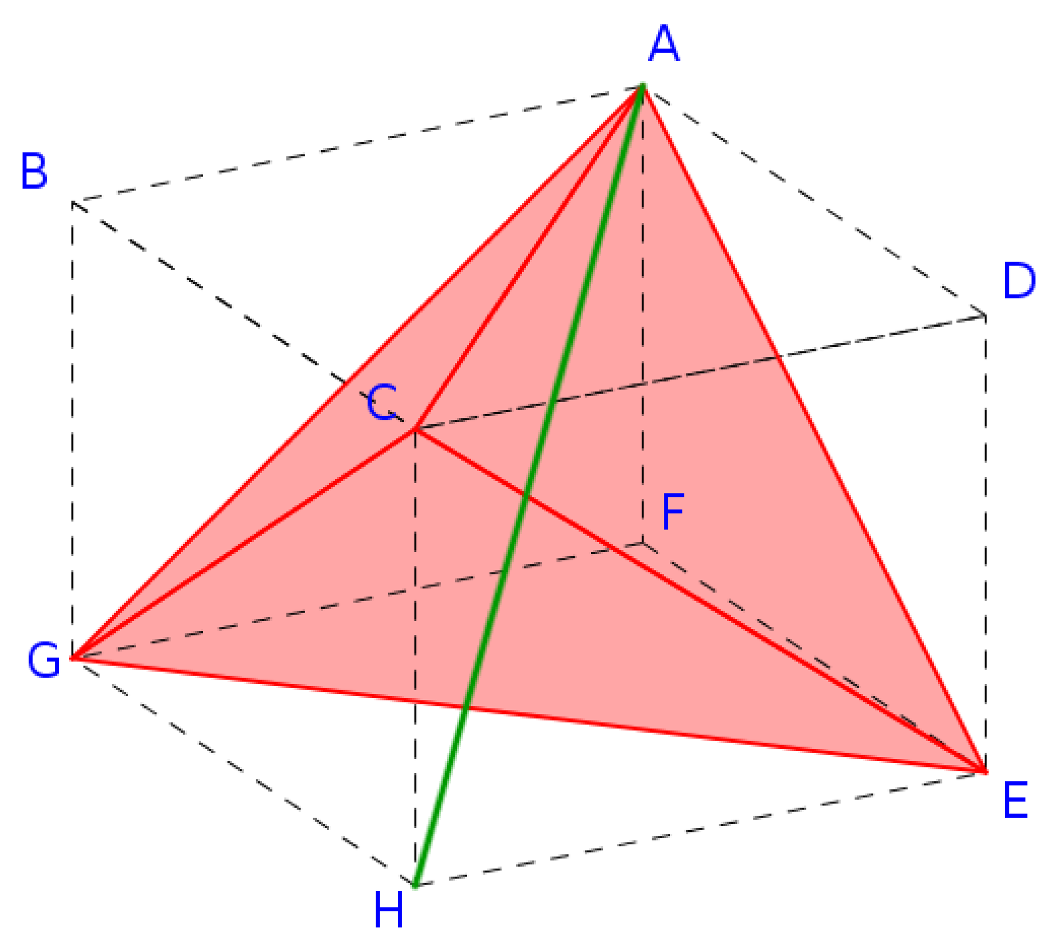
Introduction

- This is a poster investigating the Tetrahedron.
- A regular Tetrahedron is 3-dimensional, Triangular pyramid, where all faces are equilateral triangles.
- A Tetrahedron is a platonic solid. This means it is a regular convex polyhedron.
- It is constructed by congruent (identical in shape and size), regular (all angles equal and all sides equal), polygonal faces with the same number of faces meeting at each vertex.
- There are four axes of rotational symmetry which we begin to explore.
- This leads us to investigating what are the properties of tetrahedra when we give them the freedom to combine without the rotational axes.
- Finally, we examine the uniqueness of stacking tetrahedra.



References

- Elgersma, M., 2016. The Quadrahelix: A Nearly Perfect Loop Of Tetrahedra
- Hayes, B., 2012. The Science of Sticky Spheres. American Scientist
- Wikiwand. 2020. Boerdijk–Coxeter Helix
- http://www-groups.mcs.st-andrews.ac.uk/ john/geometry/Lectures/L10.html

The Reflections of a Tetrahedron

There are 6 regular reflections of a tetrahedron.

- (ABCD)-> (ABDC) Where A and B are fixed.
- (ABCD)-> (ADCB) Where A and C are fixed.
- (ABCD)-> (ACBD) Where A and D are fixed.
- (ABCD)-> (CBAD) Where B and D are fixed.
- (ABCD)-> (DBCA) Where B and C are fixed.
- (ABCD)-> (BACD) Where C and D are fixed.

There are no Irregular reflections that exist for the tetrahedrons.

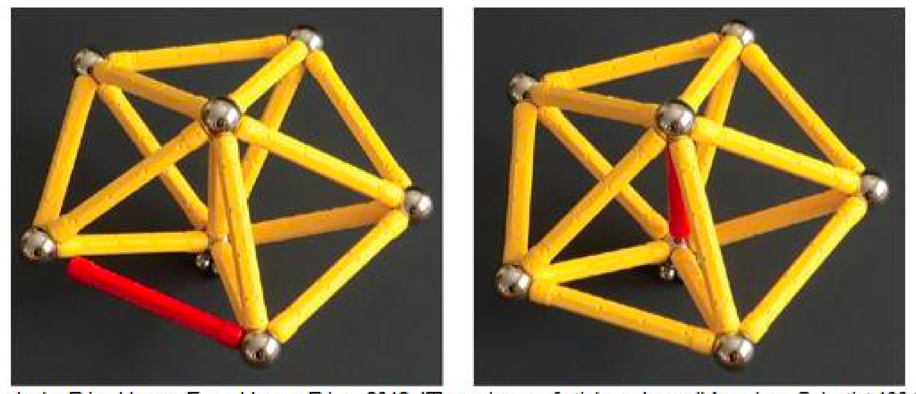
Abstract

- The group of symmetries of a Tetrahedron has 12 rotations and 12 reflections.
- This corresponds to the group S4, which is of order 4! = 24.
- There 6 'regular' rotations and 6 Rotoreflections of tetrahedrons.

Multiplication Table

R			S ∘ R		
R	Cycle	Perm.	Cycle	Perm.	S∘R
E	1	1234	(34)	1243	S
R_{12}	(12)(34)	2143	(12)	2134	SR_{12}
R_{13}	(13)(24)	3421	(1423)	3412	SR_{13}
R_{14}	(14)(23)	4321	(1324)	4312	SR_{14}
R_1	(234)	1423	(24)	1432	SR_1
R_2	(143)	3241	(13)	3214	SR_2
R_3	(124)	4132	(1234)	4123	SR_3
R_4	(132)	2314	(1432)	2341	SR_4
R_1^2	(243)	1342	(23)	1324	SR_1^2
R_2^2	(134)	4213	(14)	4231	SR_2^2
R_3^2	(142)	2431	(1342)	2413	SR_3^2
R_4^2	(123)	3124	(1243)	3142	SR_4^2

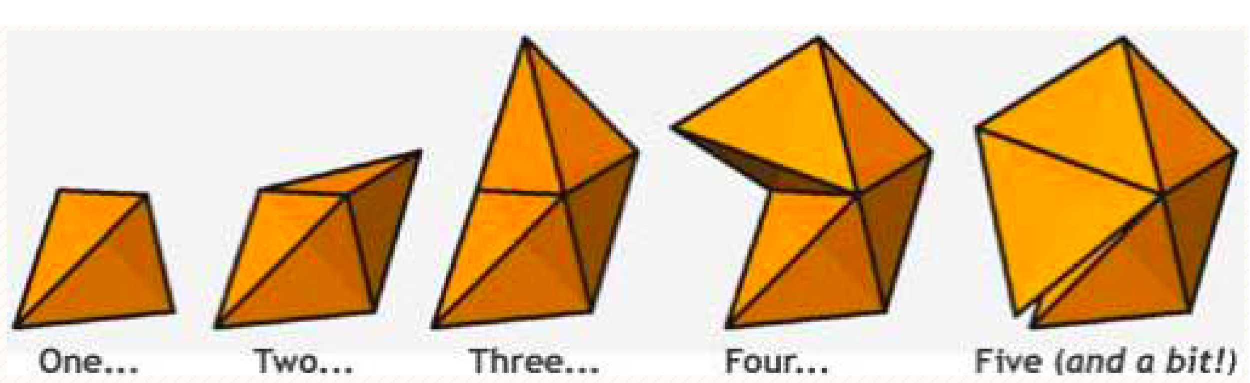
Combining Tetrahedra



Photographs by Brian Hayes. From Hayes, Brian. 2012. "The science of sticky spheres." American Scientist 100:6, p. 444.

In geometry, a tetrahedron also known as a triangular pyramid, is a polyhedron composed of four triangular faces, six straight edges, and four vertex corners. The tetrahedron is the simplest of all the ordinary convex polyhedra and the only one that has fewer than 5 faces. Five tetrahedra can almost, but not quite, meet around an edge If you try to make this happen with balls and sticks of equal length, the outer edges are not quite long enough to close the loop—and if you try to do it with five perfect tetrahedra, there is an angular gap between them. Similarly, if you have 20 perfect tetrahedra, they can almost meet around a corner. Their outer faces almost form an icosahedron, but with narrow gaps between them.

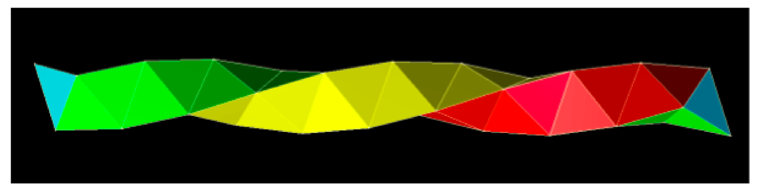
What is "Almost" doing in the Platonic world of perfection and exactitude?



Take 2 and join them face to face. . . take another and join it so they all share an edge. . . then another. . . and another. . . you are back to the beginning – full circle. . . almost!

Stacking Tetrahedra

The Boerdijk–Coxeter helix, named after H. S. M. Coxeter and A. H. Boerdijk, is a linear stacking of regular tetrahedra, arranged so that the edges of the complex that belong to only one tetrahedron form three intertwined helices. There are two chiral forms, with either clockwise or counterclockwise windings. (Art Tower Mito, Japan)



The Boerdijk–Coxeter helix can twist to infinity without ever repeating!

Aristotle's Mistake

Aristotle mistakenly thought that identical regular tetrahedrons packed together perfectly, as identical cubes do, leaving no gaps in between and filling 100 percent of the available space. They do not, and 1,800 years passed before someone pointed out that he was wrong.