Symmetries of a Cube

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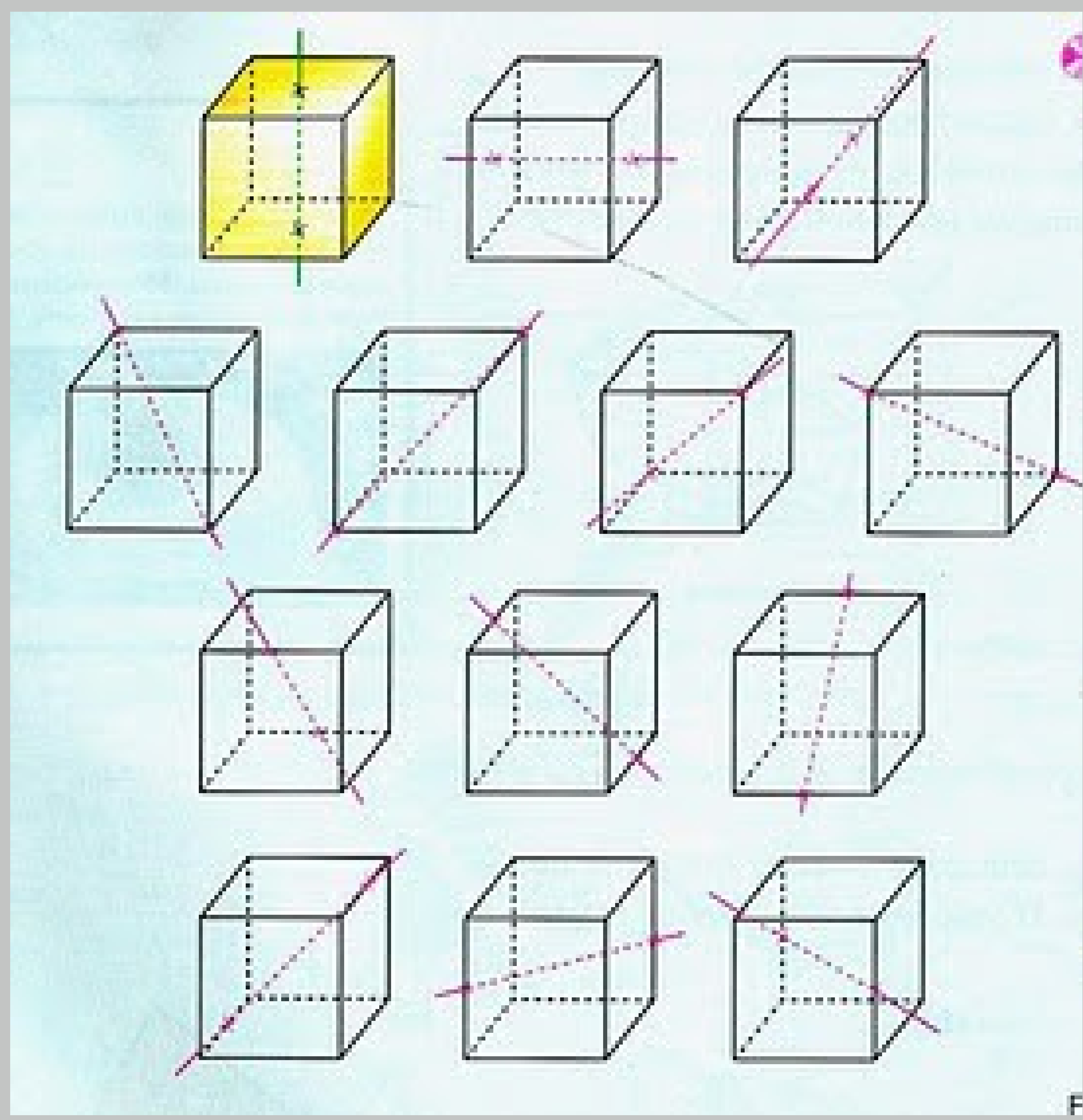
Introduction

A cube has 6 faces , 8 vertices and 12 edges and a symmetry of a cube is a permutation of its vertices that sends edges to edges.Thus there are 48 symmetries of a cube, 24 reflections and 24 rotations.
The group of rotational symmetries of a cube is isomorphic to S_4.

Rotational Symmetries

I found the easiest way to imagine the rotational symmetries of a cube was to imagine placing a spindle through 2 opposite facing edges, midpoints or faces.
Firstly lets place a spindle through a pair of opposite facing faces of the cube. There are 6 faces, 3 opposite pairs. Placing a spindle at each of these will give a symmetric rotation through 90°, 180° 270°.So each of the 3 pairs has 3 rotational symmetries, 9 in total. These are our central axis rotations.
The next pairs of opposite to consider is the edges. Placing a spindle through the midpoint of 2 opposite edges and rotating 180°will result in symmetry. 12 edges given us 6 pairs, these 6 symmetries are the diagonal midpoint symmetries.
And finally inserting a spindle through opposite pairs of corners and rotating 120°or 240°will again result in symmetric rotation of the cube. Here we have 8 corners, 4 opposite pairs with 2 symmetric rotations each, gives 8 symmetries in total about the diagonal axis.
So this gives 9 + 6 + 8= 23 rotations and of course the identity rotation which requires one either to do nothing, rotate 0 degrees or rotate 360 degrees through any of the axis explained above.

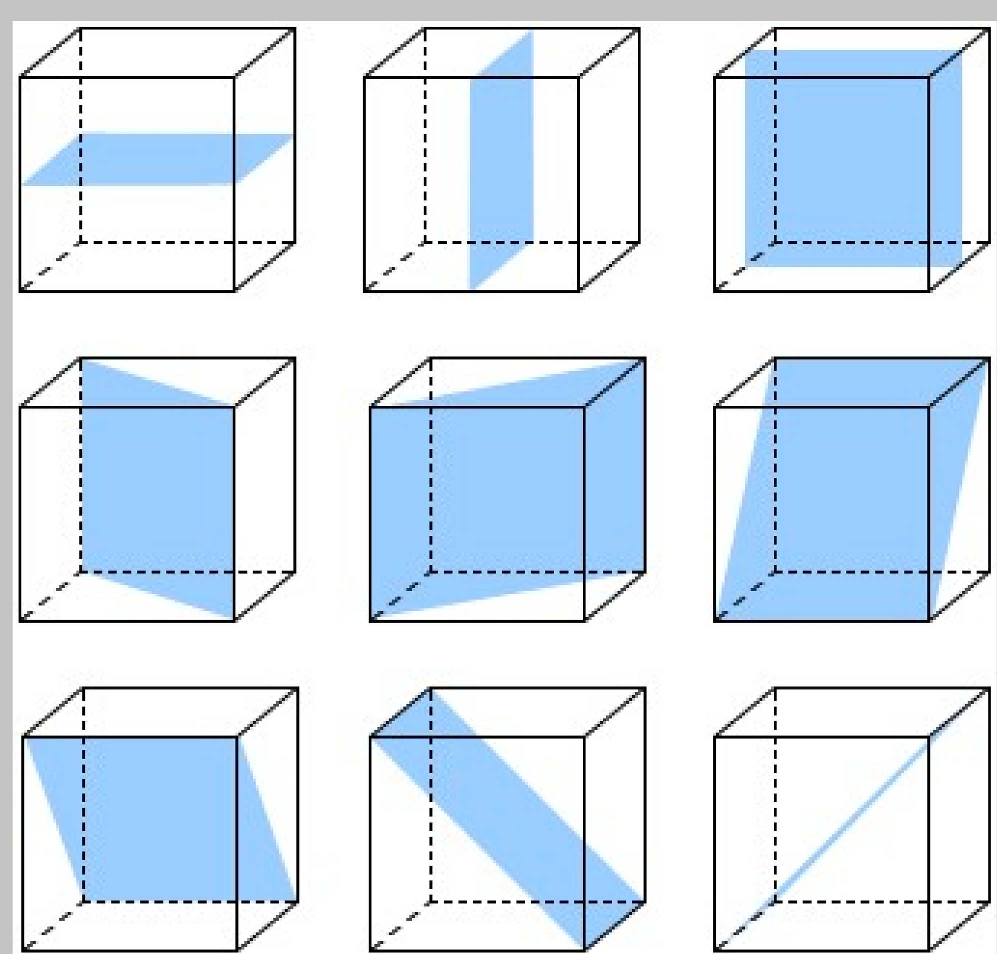
Axis of rotation



The top 3 images are the central axis, symmetry through 90°, 180° 270°rotation
2nd line are 4 diagonal axis, 120° 140°
The final 6 shown are the midpoint diagonal axis, 180°.

Reflection Symmetries

The reflection symmetries of the cube are more theoretical than the rotational ones in the sense that they are not possible to carry out in the physical 3-D world. There are 9 planes of reflection, displayed in the figure below.



Isomorphic to S_4

Each rotation is a permutation of the 8 vertices of the cube like such, 90°rotation through verticle axis gives;

$$\begin{pmatrix} 1\,2\,3\,4\,5\,6\,7\,8 \\ 2\,3\,4\,1\,6\,7\,8\,5 \end{pmatrix}, \quad (1\,2\,3\,4),\ (5\,6\,7\,8),$$

So the rotational symmetries is permutation of 2 groups of 4 objects. This is how the group of rotational symmetries of a cube is isomorphic to the symmetric group S_4

References

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