

History of Lagrange's Theorem

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Introduction

The history of Lagrange's theorem is a fascinating one, predating the idea of group theory itself. Being a now renowned fundamental, this poster project is to explore the theorem's humble beginning and the updated modern version as we know it today.



Background

On the 25th of January 1736, Giuseppe Luigi Lagrangia (his name was later anglicised to Joseph Louis Lagrange) was born in Turin, Italy. Throughout his lifespan, he made a considerable amount of contributions to mathematics, probably best known for Lagrangian mechanics as well as the theorem we will be diving into in a bit more detail.

Lagrange's Original Theorem

Lagrange's initial goal was to solve for the roots of a 5th degree polynomial, with no advances being made since the 1500's. His theorem was that:

If a function $f(x_1, x_2, \dots, x_n)$ of n variables is acted on by all $n!$ permutations of the variables and those permuted functions take on only r distinct values, then r is a divisor of $n!$

The Modern Application of Lagrange's Theorem

Since then, a lot of work has been done since we've noticed how applicable the first theorem could be! Cauchy and Galois built further upon it, applying it to what we now know as group theory!

The modern understanding of Lagrange's theorem is:

If G is a finite group, and H is a subgroup of G , then the order of G is divisible by the order of H .

An example of Lagrange's theorem at work would be for the group of 6th roots of unity, where each subgroup has order of 2 or 3, and also divides into 6

