

The Group of Symmetries of the Cube

Seán Tynan Luke Finn

Introduction

This is a poster about the group of symmetries of a cube. A cube is a 3D shape with 6 square faces, 8 vertices and 12 edges. There is a total of 48 symmetries of the cube. Comprising of 24 rotational symmetries and 24 reflections.

Reflection Symmetries

There are a total of 24 reflection symmetries of the cube and these are consisting of:

- ▶ 15 turn reflections.
- ▶ 9 plane reflections.

Plane Reflections

The cube has 9 reflection planes which are:

- ▶ 3 planes lie parallel to the side squares and go through the centre.
- ▶ 6 planes go through opposite edges and two body diagonals. They divide the cube into prisms.

Turn Reflections

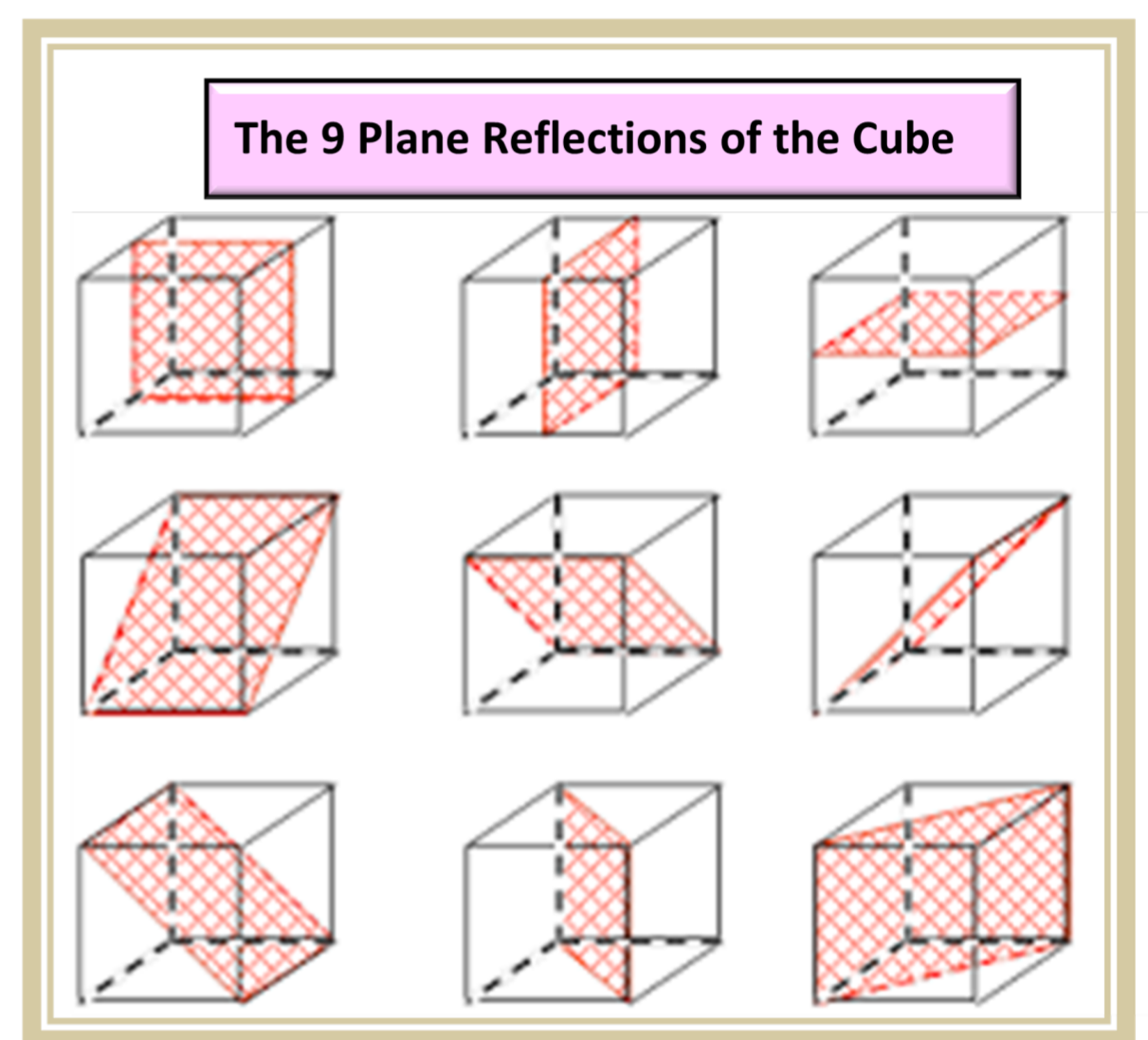
There are 3 axes which exists from the center of one face to the center of the opposite face and they can each be rotated 4 times. These degrees of rotation are 90° , 180° and 270° , not counting the identity. However since a 180° turn reflection is actually the antipodal symmetry, there are actually 6 turn reflections. Consisting of 3 each for the 90° and 270° rotations.

There are 4 axes which exists from a vertex to the diagonally opposing vertex and they can each be rotated 3 times. These degrees of rotation are 120° and 240° , not counting the identity. Therefore there are 8 of these turn reflections.

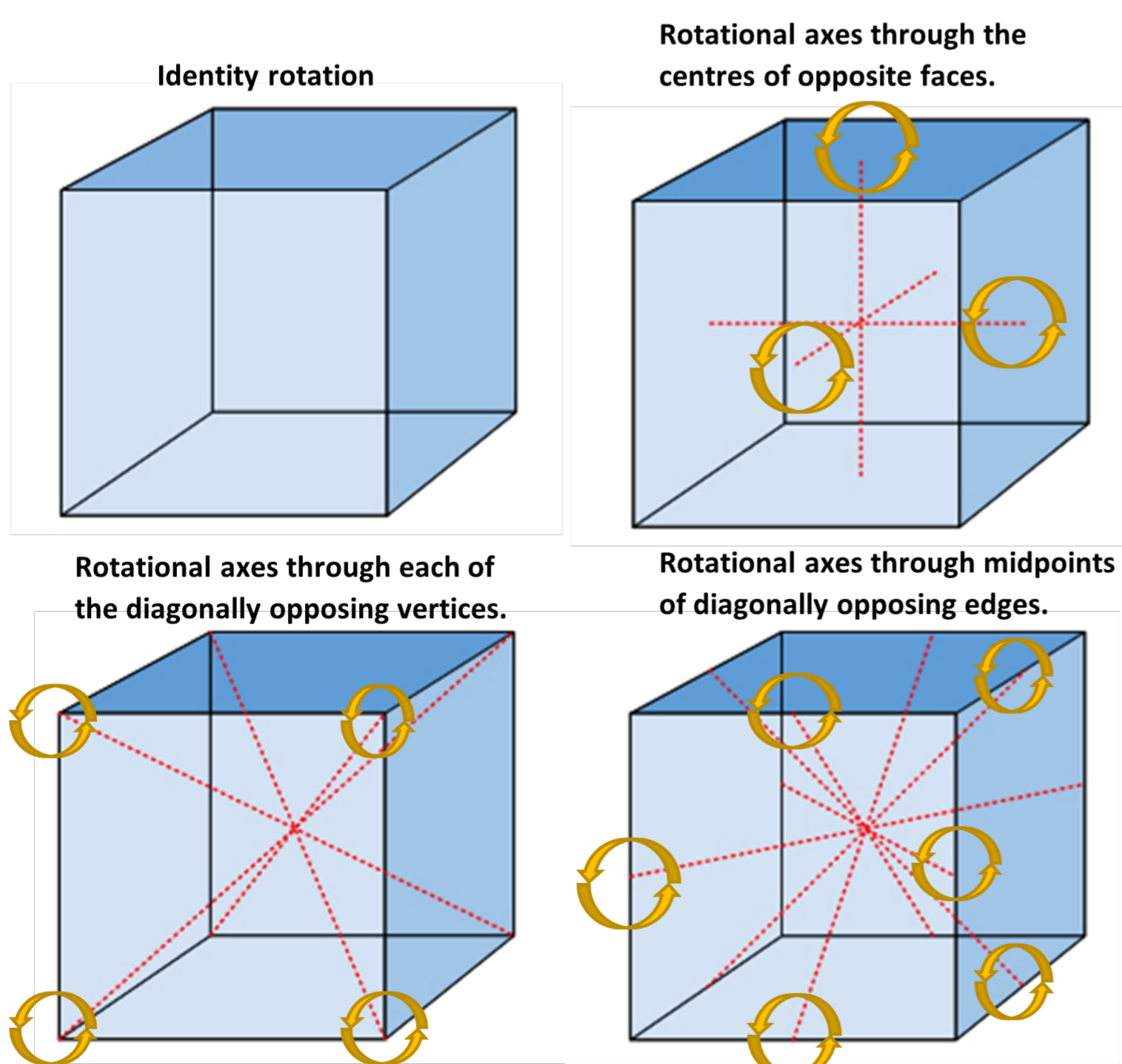
There are 6 axes which exists from the midpoint of one edge to the midpoint of the diagonally opposing edge and they can each be rotated twice. Again, not counting the identity, this degree of rotation is 180° . But since each of these 180° rotations are really the antipodal symmetry, there are no turn reflections for this axis either.

Finally we have the antipodal symmetry which was not counted in any of the above turn reflections. This is the 180° turn reflection. This is 1 turn reflection and combined with the previous 6 and 8 turn reflections stated above, brings the total to 15.

Reflection Symmetries Graphic



Rotation Axes Graphics



Rotational Symmetries

There is a total of 24 rotational symmetries of the cube, all of which are anti-clockwise. These consist of:

- ▶ 9 rotations about lines through the centres of opposite faces. There is a total of 3 axes of rotation, each of which has 3 rotations. Consisting of 90° , 180° , 270° .
- ▶ 8 rotations about lines through diagonally opposing vertices. There is a total of 4 axes of rotation, each of which has 2 rotations. Consisting of 120° , 240° .
- ▶ 6 rotations about lines through midpoints of diagonally opposing edges. There is a total of 6 axes of rotation, each of which has 1 rotation. This is the rotation through 180° .
- ▶ Identity which is a rotation 0° or 360° through any of these axes.

References

Rotation images - <https://i0.wp.com/peterjamesthomas.com/wp-content/uploads/2016/08/rotational-group-of-a-cube.jpg?ssl=1>

Reflections -

<http://jwilson.coe.uga.edu/EMAT6680Fa06/Sexton/NCTMThreeDimensionalGeometry/SymmetryofaCube.html>