Symmetries in Nature

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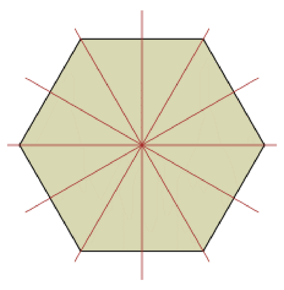
Introduction

As we observe our environment and our surroundings, we find patterns are abundant in the natural world. Patterns are visible regularities which sometimes can be modelled mathematically. From Early Greek philosophers to us now, humans studied patterns attempting to explain the order in nature. The beauty of rotational symmetry of a snowflake, the satisfying glided pattern of snake skin, the resourcefulness of the icosahedron for viruses all highlight nature's inherent propensity to form patterns. This poster attempts to link the beauty and resourcefulness of nature and mathematics in regards to symmetry.

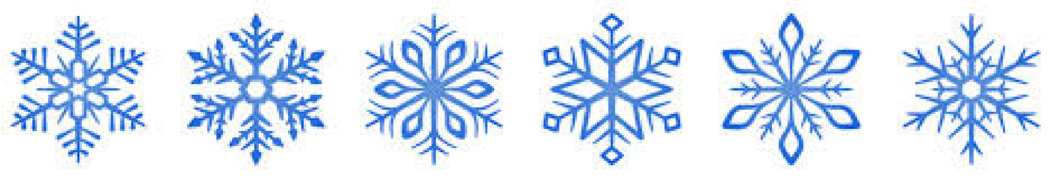
Dihedral Groups

A dihedral group is the group of symmetries of a regular polygon, the group D2n consists of n rotations and n reflections.

There are plenty of examples of such groups in nature, the most common is the group D12 - the symmetries of a hexagon. This group has six reflections as shown below, and 6 rotations: id, R60, R120, R180, R240, R300



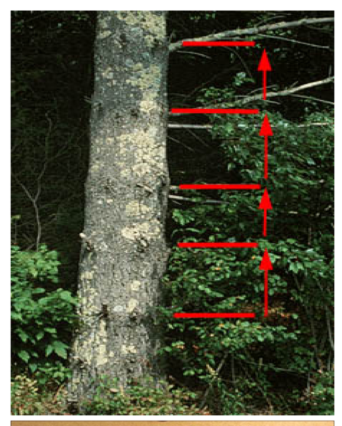
We see snowflakes have this hexagonal shape and the symmetries and rotations of D12. Honeycombs are another example, and even a close look at a dragonfly's eye shows that it is a collection of tiny lenses - all of which are hexagonal shaped. D12 is one of the many dihedral groups we can see throughout nature.



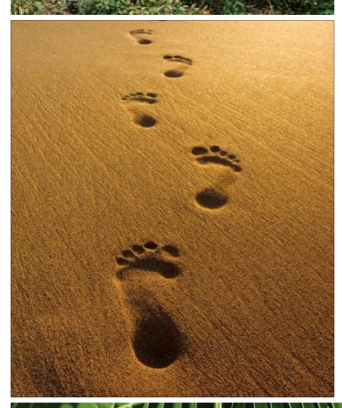
Frieze Patterns

A Frieze, or Strip Pattern contain either all or some of the following types of symmetries: Translations, Horizontal mirror reflections, Vertical mirror reflections, Rotations, Glide reflection. These symmetries form a group of seven distinct patterns: T, TR, TV, TG, TRVG, TGH and TRGHV.

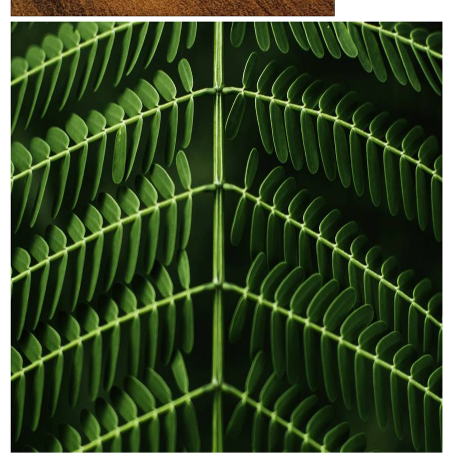
This is a picture of White Pine with some of its branches fallen off. See how each year, the tree grows new branches above each other. The white pine exhibits Translation symmetry.



Here are pictures of footprints, they are human footprints in sand. See how this forms a Translation and Glide Reflection. The right hand-side picture is a set of bear tracks. See how this is also TG symmetry.

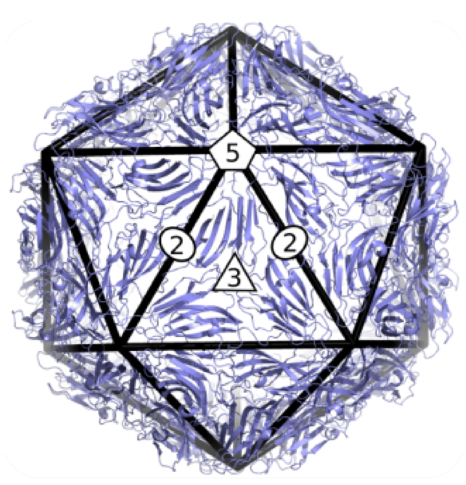


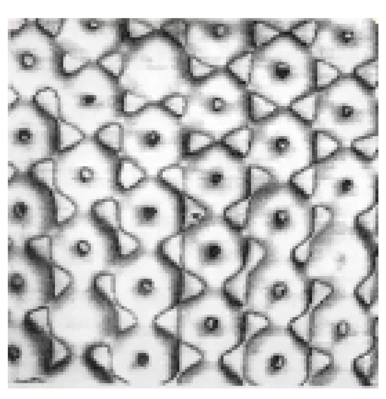
Below the footprints, is a leaf from the Mimosa tree. It displays TGH (translation and horizontal mirroring and a glide symmetry by default) symmetry.

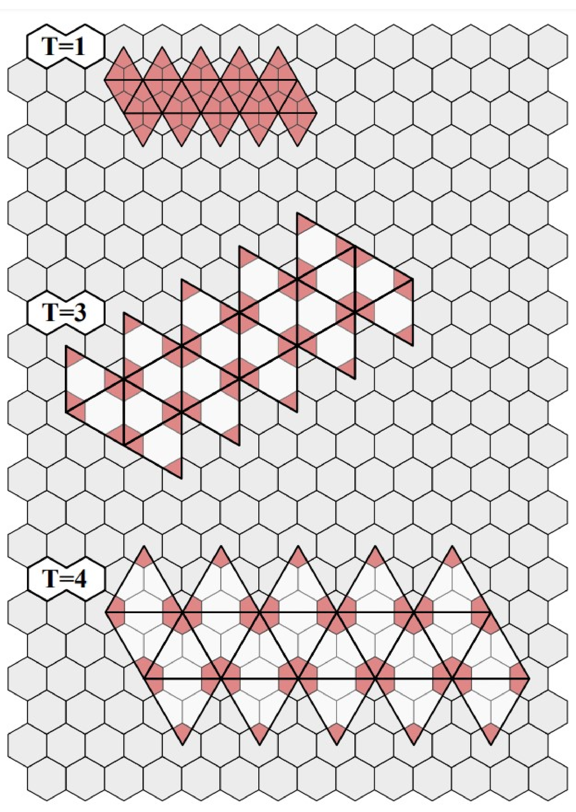


Most snakes have TRGHV symmetry (Translation, Horizontal and Vertical Reflection and Rotation 180). This is a picture of a copperhead snake and the snake skin.

Symmetries in 3D







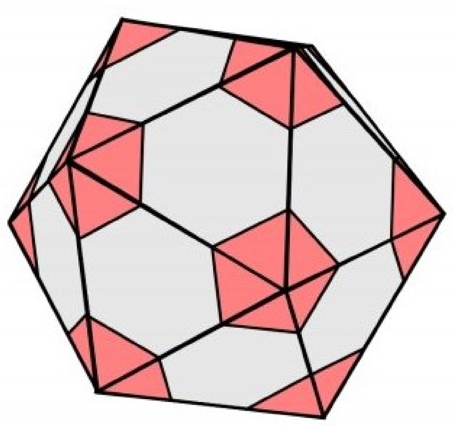
Virology

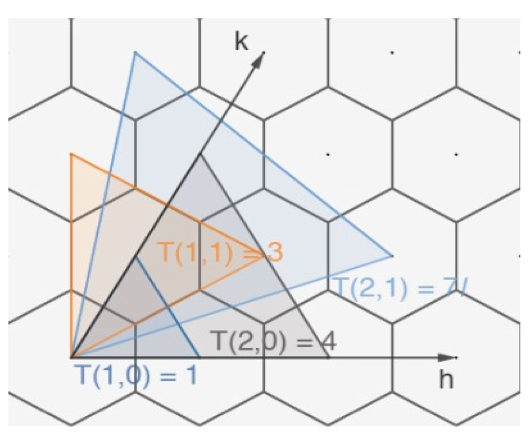
3-Dimensional objects with symmetric qualities are also evident in nature. Viruses are perhaps a surprising example of such symmetry. Viruses are protected by a protein container, called a "viral capsid". Viral capsids transport genetic material into a host, and thus hijack their hosts machinery to reproduce. For the majority of viruses, these capsids have icosahedral symmetry. It is a large group of symmetries, with order 120. 60 of these are rotations. The axes of symmetry are visible on the diagram. There is 31 in all. 15 axes through edges, 10 through the centres of faces, and 6 through vertices.

Why Icosahedrons?

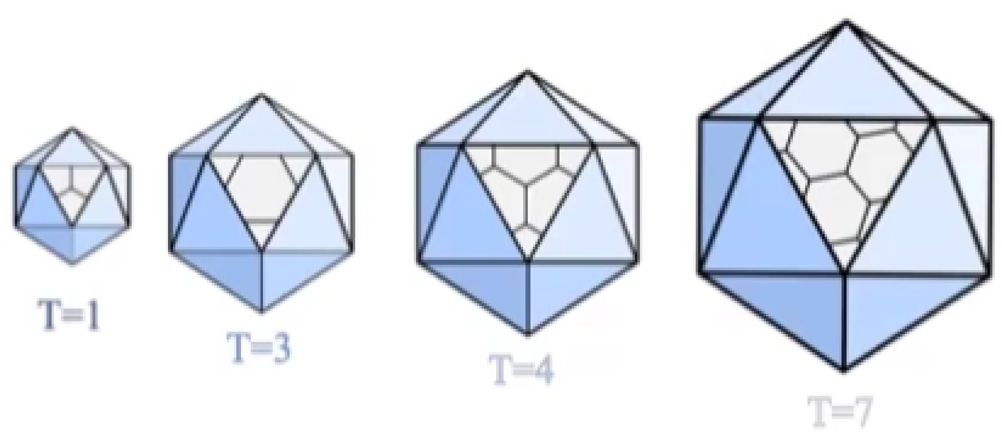
Viruses build these icosahedral capsids for reasons of genetic economy. The icosahedron consists of 20 triangular faces, and is the largest platonic solid. This makes it a relatively simple shape to produce. Viruses are extremely small, and so they have to be efficient with every piece of their genetic code. Using containers with icosahedral symmetry allows viruses to create a sufficiently large container to hold their genetic material, while minimising the amount of genetic code required.

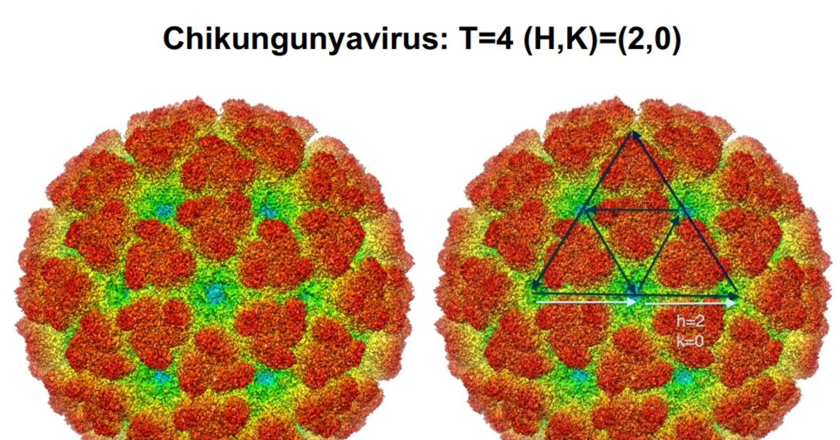
The biologists Caspar and Klug used the icosahedral nature of viruses in the 1960s to create a system known as the triangulation-number or "T-number" series. This series is used to describe different viral architectures. They were inspired by the lattice-like structure of the surface of this virus (middle left), with somewhat hexagonal groups of 6 proteins (hexamers) composing most of the shell, and groups of 5 proteins (pentamers) at the vertices. They overlayed the net of the icosahedron over a grid of hexagons, such that the vertex of each triangular face was at the centre of a hexagon. They were then able to produce many more viral capsids with different protein layouts by rescaling and rotating the net, as seen below. All of these capsids retain the symmetric properties of the icosahedron.





$T(h,k) = h^2 + hk + k^2$





References

Prof. Reidun Twarock - "Geometry: A New Weapon in the Fight Against Viruses" - London Mathematical Society