

Finite Simple Groups and The Monster

Michael McGloin

What Are Finite Simple Groups?

Recall that a subgroup H of G is **normal** if :

$$gH = Hg \text{ for any } g \in G$$

Finite simple groups are finite groups whose only normal subgroups are the identity element and the group itself. Finite simple groups are like the "elements" of groups. Every group can be broken down into these finite simple groups by looking at so called factor groups of **composition series**

What is a Composition Series?

To understand a composition series we will first look at the definition of a subnormal series. A **subnormal series** of a group G is a sequence of subgroups of G whereby each subsequent subgroup is normal to the next one.

$$G = H_1 \trianglelefteq H_2 \trianglelefteq H_3 \dots \trianglelefteq H_n$$

Note it is not the case that each H_i be a normal subgroup of G but only to H_{i+1} .

A subnormal series :

$$\{e\} = H_1 \trianglelefteq H_2 \trianglelefteq H_3 \dots \trianglelefteq H_n = G$$

is a composition series if each factor H_{i+1}/H_i is a simple group. Every group G has a composition series.

Jordan-Hölder Theorem

Any two composition series of a group G are equivalent. Therefore every group having a composition series determines a unique list of finite simple groups and it is in this sense that the finite simple groups are the building blocks of groups !

Periodic Table of Finite Simple Groups

The Periodic Table Of Finite Simple Groups

Dynkin Diagrams of Simple Lie Algebras

$0, C_2, Z_3$	$A_1(2)$	$A_2(3)$	$A_3(4)$	$A_4(5)$	$A_5(6)$	$A_6(7)$	$A_7(8)$	$A_8(9)$	$A_9(10)$	$A_{10}(11)$	$A_{11}(12)$	$A_{12}(13)$	$A_{13}(14)$	$A_{14}(15)$	$A_{15}(16)$	$A_{16}(17)$	$A_{17}(18)$	$A_{18}(19)$	$A_{19}(20)$	$A_{20}(21)$	$A_{21}(22)$	$A_{22}(23)$	$A_{23}(24)$	$A_{24}(25)$	$A_{25}(26)$	$A_{26}(27)$	$A_{27}(28)$	$A_{28}(29)$	$A_{29}(30)$	$A_{30}(31)$	$A_{31}(32)$	$A_{32}(33)$	$A_{33}(34)$	$A_{34}(35)$	$A_{35}(36)$	$A_{36}(37)$	$A_{37}(38)$	$A_{38}(39)$	$A_{39}(40)$	$A_{40}(41)$	$A_{41}(42)$	$A_{42}(43)$	$A_{43}(44)$	$A_{44}(45)$	$A_{45}(46)$	$A_{46}(47)$	$A_{47}(48)$	$A_{48}(49)$	$A_{49}(50)$	$A_{50}(51)$	$A_{51}(52)$	$A_{52}(53)$	$A_{53}(54)$	$A_{54}(55)$	$A_{55}(56)$	$A_{56}(57)$	$A_{57}(58)$	$A_{58}(59)$	$A_{59}(60)$	$A_{60}(61)$	$A_{61}(62)$	$A_{62}(63)$	$A_{63}(64)$	$A_{64}(65)$	$A_{65}(66)$	$A_{66}(67)$	$A_{67}(68)$	$A_{68}(69)$	$A_{69}(70)$	$A_{70}(71)$	$A_{71}(72)$	$A_{72}(73)$	$A_{73}(74)$	$A_{74}(75)$	$A_{75}(76)$	$A_{76}(77)$	$A_{77}(78)$	$A_{78}(79)$	$A_{79}(80)$	$A_{80}(81)$	$A_{81}(82)$	$A_{82}(83)$	$A_{83}(84)$	$A_{84}(85)$	$A_{85}(86)$	$A_{86}(87)$	$A_{87}(88)$	$A_{88}(89)$	$A_{89}(90)$	$A_{90}(91)$	$A_{91}(92)$	$A_{92}(93)$	$A_{93}(94)$	$A_{94}(95)$	$A_{95}(96)$	$A_{96}(97)$	$A_{97}(98)$	$A_{98}(99)$	$A_{99}(100)$	$A_{100}(101)$	$A_{101}(102)$	$A_{102}(103)$	$A_{103}(104)$	$A_{104}(105)$	$A_{105}(106)$	$A_{106}(107)$	$A_{107}(108)$	$A_{108}(109)$	$A_{109}(110)$	$A_{110}(111)$	$A_{111}(112)$	$A_{112}(113)$	$A_{113}(114)$	$A_{114}(115)$	$A_{115}(116)$	$A_{116}(117)$	$A_{117}(118)$	$A_{118}(119)$	$A_{119}(120)$	$A_{120}(121)$	$A_{121}(122)$	$A_{122}(123)$	$A_{123}(124)$	$A_{124}(125)$	$A_{125}(126)$	$A_{126}(127)$	$A_{127}(128)$	$A_{128}(129)$	$A_{129}(130)$	$A_{130}(131)$	$A_{131}(132)$	$A_{132}(133)$	$A_{133}(134)$	$A_{134}(135)$	$A_{135}(136)$	$A_{136}(137)$	$A_{137}(138)$	$A_{138}(139)$	$A_{139}(140)$	$A_{140}(141)$	$A_{141}(142)$	$A_{142}(143)$	$A_{143}(144)$	$A_{144}(145)$	$A_{145}(146)$	$A_{146}(147)$	$A_{147}(148)$	$A_{148}(149)$	$A_{149}(150)$	$A_{150}(151)$	$A_{151}(152)$	$A_{152}(153)$	$A_{153}(154)$	$A_{154}(155)$	$A_{155}(156)$	$A_{156}(157)$	$A_{157}(158)$	$A_{158}(159)$	$A_{159}(160)$	$A_{160}(161)$	$A_{161}(162)$	$A_{162}(163)$	$A_{163}(164)$	$A_{164}(165)$	$A_{165}(166)$	$A_{166}(167)$	$A_{167}(168)$	$A_{168}(169)$	$A_{169}(170)$	$A_{170}(171)$	$A_{171}(172)$	$A_{172}(173)$	$A_{173}(174)$	$A_{174}(175)$	$A_{175}(176)$	$A_{176}(177)$	$A_{177}(178)$	$A_{178}(179)$	$A_{179}(180)$	$A_{180}(181)$	$A_{181}(182)$	$A_{182}(183)$	$A_{183}(184)$	$A_{184}(185)$	$A_{185}(186)$	$A_{186}(187)$	$A_{187}(188)$	$A_{188}(189)$	$A_{189}(190)$	$A_{190}(191)$	$A_{191}(192)$	$A_{192}(193)$	$A_{193}(194)$	$A_{194}(195)$	$A_{195}(196)$	$A_{196}(197)$	$A_{197}(198)$	$A_{198}(199)$	$A_{199}(200)$	$A_{200}(201)$	$A_{201}(202)$	$A_{202}(203)$	$A_{203}(204)$	$A_{204}(205)$	$A_{205}(206)$	$A_{206}(207)$	$A_{207}(208)$	$A_{208}(209)$	$A_{209}(210)$	$A_{210}(211)$	$A_{211}(212)$	$A_{212}(213)$	$A_{213}(214)$	$A_{214}(215)$	$A_{215}(216)$	$A_{216}(217)$	$A_{217}(218)$	$A_{218}(219)$	$A_{219}(220)$	$A_{220}(221)$	$A_{221}(222)$	$A_{222}(223)$	$A_{223}(224)$	$A_{224}(225)$	$A_{225}(226)$	$A_{226}(227)$	$A_{227}(228)$	$A_{228}(229)$	$A_{229}(230)$	$A_{230}(231)$	$A_{231}(232)$	$A_{232}(233)$	$A_{233}(234)$	$A_{234}(235)$	$A_{235}(236)$	$A_{236}(237)$	$A_{237}(238)$	$A_{238}(239)$	$A_{239}(240)$	$A_{240}(241)$	$A_{241}(242)$	$A_{242}(243)$	$A_{243}(244)$	$A_{244}(245)$	$A_{245}(246)$	$A_{246}(247)$	$A_{247}(248)$	$A_{248}(249)$	$A_{249}(250)$	$A_{250}(251)$	$A_{251}(252)$	$A_{252}(253)$	$A_{253}(254)$	$A_{254}(255)$	$A_{255}(256)$	$A_{256}(257)$	$A_{257}(258)$	$A_{258}(259)$	$A_{259}(260)$	$A_{260}(261)$	$A_{261}(262)$	$A_{262}(263)$	$A_{263}(264)$	$A_{264}(265)$	$A_{265}(266)$	$A_{266}(267)$	$A_{267}(268)$	$A_{268}(269)$	$A_{269}(270)$	$A_{270}(271)$	$A_{271}(272)$	$A_{272}(273)$	$A_{273}(274)$	$A_{274}(275)$	$A_{275}(276)$	$A_{276}(277)$	$A_{277}(278)$	$A_{278}(279)$	$A_{279}(280)$	$A_{280}(281)$	$A_{281}(282)$	$A_{282}(283)$	$A_{283}(284)$	$A_{284}(285)$	$A_{285}(286)$	$A_{286}(287)$	$A_{287}(288)$	$A_{288}(289)$	$A_{289}(290)$	$A_{290}(291)$	$A_{291}(292)$	$A_{292}(293)$	$A_{293}(294)$	$A_{294}(295)$	$A_{295}(296)$	$A_{296}(297)$	$A_{297}(298)$	$A_{298}(299)$	$A_{299}(300)$	$A_{300}(301)$	$A_{301}(302)$	$A_{302}(303)$	$A_{303}(304)$	$A_{304}(305)$	$A_{305}(306)$	$A_{306}(307)$	$A_{307}(308)$	$A_{308}(309)$	$A_{309}(310)$	$A_{310}(311)$	$A_{311}(312)$	$A_{312}(313)$	$A_{313}(314)$	$A_{314}(315)$	$A_{315}(316)$	$A_{316}(317)$	$A_{317}(318)$	$A_{318}(319)$	$A_{319}(320)$	$A_{320}(321)$	$A_{321}(322)$	$A_{322}(323)$	$A_{323}(324)$	$A_{324}(325)$	$A_{325}(326)$	$A_{326}(327)$	$A_{327}(328)$	$A_{328}(329)$	$A_{329}(330)$	$A_{330}(331)$	$A_{331}(332)$	$A_{332}(333)$	$A_{333}(334)$	$A_{334}(335)$	$A_{335}(336)$	$A_{336}(337)$	$A_{337}(338)$	$A_{338}(339)$	$A_{339}(340)$	$A_{340}(341)$	$A_{341}(342)$	$A_{342}(343)$	$A_{343}(344)$	$A_{344}(345)$	$A_{345}(346)$	$A_{346}(347)$	$A_{347}(348)$	$A_{348}(349)$	$A_{349}(350)$	$A_{350}(351)$	$A_{351}(352)$	$A_{352}(353)$	$A_{353}(354)$	$A_{354}(355)$	$A_{355}(356)$	$A_{356}(357)$	$A_{357}(358)$	$A_{358}(359)$	$A_{359}(360)$	$A_{360}(361)$	$A_{361}(362)$	$A_{362}(363)$	$A_{363}(364)$	$A_{364}(365)$	$A_{365}(366)$	$A_{366}(367)$	$A_{367}(368)$	$A_{368}(369)$	$A_{369}(370)$	$A_{370}(371)$	$A_{371}(372)$	$A_{372}(373)$	$A_{373}(374)$	$A_{374}(375)$	$A_{375}(376)$	$A_{376}(377)$	$A_{377}(378)$	$A_{378}(379)$	$A_{379}(380)$	$A_{380}(381)$	$A_{381}(382)$	$A_{382}(383)$	$A_{383}(384)$	$A_{384}(385)$	$A_{385}(386)$	$A_{386}(387)$	$A_{387}(388)$	$A_{388}(389)$	$A_{389}(390)$	$A_{390}(391)$	$A_{391}(392)$	$A_{392}(393)$	$A_{393}(394)$	$A_{394}(395)$	$A_{395}(396)$	$A_{396}(397)$	$A_{397}(398)$	$A_{398}(399)$	$A_{399}(400)$	$A_{400}(401)$	$A_{401}(402)$	$A_{402}(403)$	$A_{403}(404)$	$A_{404}(405)$	$A_{405}(406)$	$A_{406}(407)$	$A_{407}(408)$	$A_{408}(409)$	$A_{409}(410)$	$A_{410}(411)$	$A_{411}(412)$	$A_{412}(413)$	$A_{413}(414)$	$A_{414}(415)$	$A_{415}(416)$	$A_{416}(417)$	$A_{417}(418)$	$A_{418}(419)$	$A_{419}(420)$	$A_{420}(421)$	$A_{421}(422)$	$A_{422}(423)$	$A_{423}(424)$	$A_{424}(425)$	$A_{425}(426)$	$A_{426}(427)$	$A_{427}(428)$	$A_{428}(429)$	$A_{429}(430)$	$A_{430}(431)$	$A_{431}(432)$	$A_{432}(433)$	$A_{433}(434)$	$A_{434}(435)$	$A_{435}(436)$	$A_{436}(437)$	$A_{437}(438)$	$A_{438}(439)$	$A_{439}(440)$	$A_{440}(441)$	$A_{441}(442)$	$A_{442}(443)$	$A_{443}(444)$	$A_{444}(445)$	$A_{445}(446)$	$A_{446}(447)$	$A_{447}(448)$	$A_{448}(449)$	$A_{449}(450)$	$A_{450}(451)$	$A_{451}(452)$	$A_{452}(453)$	$A_{453}(454)$	$A_{454}(455)$	$A_{455}(456)$	$A_{456}(457)$	$A_{457}(458)$	$A_{458}(459)$	$A_{459}(460)$	$A_{460}(461)$	$A_{461}(462)$	$A_{462}(463)$	$A_{463}(464)$	$A_{464}(465)$	$A_{465}(466)$	$A_{466}(467)$	$A_{467}(468)$	$A_{468}(469)$	$A_{469}(470)$	$A_{470}(471)$	$A_{471}(472)$	$A_{472}(473)$	$A_{473}(474)$	$A_{474}(475)$	$A_{475}(476)$	$A_{476}(477)$	$A_{477}(478)$	$A_{478}(479)$	$A_{479}(480)$	$A_{480}(481)$	$A_{481}(482)$	$A_{482}(483)$	$A_{483}(484)$	$A_{484}(485)$	$A_{485}(486)$	$A_{486}(487)$	$A_{487}(488)$	$A_{488}(489)$	$A_{489}(490)$	$A_{490}(491)$	$A_{491}(492)$	$A_{492}(493)$	$A_{493}(494)$	$A_{494}(495)$	$A_{495}(496)$	$A_{496}(497)$	$A_{497}(498)$	$A_{498}(499)$	$A_{499}(500)$	$A_{500}(501)$	$A_{501}(502)$	$A_{502}(503)$	$A_{503}(504)$	$A_{504}(505)$	$A_{505}(506)$	$A_{506}(507)$	$A_{507}(508)$	$A_{508}(509)$	$A_{509}(510)$	$A_{510}(511)$	$A_{511}(512)$	$A_{512}(513)$	$A_{513}(514)$	$A_{514}(515)$	$A_{515}(516)$	$A_{516}(517)$	$A_{517}(518)$	$A_{518}(519)$	$A_{519}(520)$	$A_{520}(521)$	$A_{521}(522)$	$A_{522}(523)$	$A_{523}(524)$	$A_{524}(525)$	$A_{525}(526)$	$A_{526}(527)$	$A_{527}(528)$	$A_{528}(529)$	$A_{529}(530)$	$A_{530}(531)$	$A_{531}(532)$	$A_{532}(533)$	$A_{533}(534)$	$A_{534}(535)$	$A_{535}(536)$	$A_{536}(537)$	$A_{537}(538)$	$A_{538}(539)$	$A_{539}(540)$	$A_{540}(541)$	$A_{541}(542)$	$A_{542}(543)$	$A_{543}(544)$	$A_{544}(545)$	$A_{545}(546)$	$A_{546}(547)$	$A_{547}(548)$	$A_{548}(549)$	$A_{549}(550)$	$A_{550}(551)$	$A_{551}(552)$	$A_{552}(553)$	$A_{553}(554)$	$A_{554}(555)$	$A_{555}(556)$	$A_{556}(557)$	$A_{557}(558)$	$A_{558}(559)$	$A_{559}(560)$	$A_{560}(561)$	$A_{561}(562)$	$A_{562}(563)$	$A_{563}(564)$	$A_{564}(565)$	$A_{565}(566)$	$A_{566}(567)$	$A_{567}(568)$	$A_{568}(569)$	$A_{569}(570)$	$A_{570}(571)$	$A_{571}(572)$	$A_{572}(573)$	$A_{573}(574)$	$A_{574}(575)$	$A_{575}(576)$	$A_{576}(577)$	$A_{577}(578)$	$A_{578}(579)$	$A_{579}(580)$	$A_{580}(581)$	$A_{581}(582)$	$A_{582}(583)$	$A_{583}(584)$	$A_{584}(585)$	$A_{585}(586)$	$A_{586}(587)$	$A_{587}(588)$	$A_{588}(589)$	$A_{589}(590)$	$A_{590}(591)$	$A_{591}(592)$	$A_{592}(593)$	$A_{593}(594)$	$A_{594}(595)$	$A_{595}(596)$	$A_{596}(597)$	$A_{597}(598)$	$A_{598}(599)$	$A_{599}(600)$	$A_{600}(601)$	$A_{601}(602)$	$A_{602}(603)$	$A_{603}(604)$	$A_{604}(605)$	$A_{605}(606)$	$A_{606}(607)$	$A_{607}(608)$	$A_{608}(609)$	$A_{609}(610)$	$A_{610}(611)$	$A_{611}(612)$	$A_{612}(613)$	$A_{613}(614)$	$A_{614}(615)$	$A_{615}(616)$	$A_{616}(617)$	$A_{617}(618)$	$A_{618}(619)$	$A_{619}(620)$	$A_{620}(621)$	$A_{621}(622)$	$A_{622}(623)$	$A_{623}(624)$	$A_{624}(625)$	$A_{625}(626)$	$A_{626}(627)$	$A_{627}(628)$	$A_{628}(629)$	$A_{629}(630)$	$A_{630}(631)$	$A_{631}(632)$	$A_{632}(633)$	$A_{633}(634)$	$A_{634}(635)$	$A_{635}(636)$	
---------------	----------	----------	----------	----------	----------	----------	----------	----------	-----------	--------------	--------------	--------------	--------------	--------------	--------------	--------------	--------------	--------------	--------------	--------------	--------------	--------------	--------------	--------------	--------------	--------------	--------------	--------------	--------------	--------------	--------------	--------------	--------------	--------------	--------------	--------------	--------------	--------------	--------------	--------------	--------------	--------------	--------------	--------------	--------------	--------------	--------------	--------------	--------------	--------------	--------------	--------------	--------------	--------------	--------------	--------------	--------------	--------------	--------------	--------------	--------------	--------------	--------------	--------------	--------------	--------------	--------------	--------------	--------------	--------------	--------------	--------------	--------------	--------------	--------------	--------------	--------------	--------------	--------------	--------------	--------------	--------------	--------------	--------------	--------------	--------------	--------------	--------------	--------------	--------------	--------------	--------------	--------------	--------------	--------------	--------------	--------------	--------------	---------------	----------------	----------------	----------------	----------------	----------------	----------------	----------------	----------------	----------------	----------------	----------------	----------------	----------------	----------------	----------------	----------------	----------------	----------------	----------------	----------------	----------------	----------------	----------------	----------------	----------------	----------------	----------------	----------------	----------------	----------------	----------------	----------------	----------------	----------------	----------------	----------------	----------------	----------------	----------------	----------------	----------------	----------------	----------------	----------------	----------------	----------------	----------------	----------------	----------------	----------------	----------------	----------------	----------------	----------------	----------------	----------------	----------------	----------------	----------------	----------------	----------------	----------------	----------------	----------------	----------------	----------------	----------------	----------------	----------------	----------------	----------------	----------------	----------------	----------------	----------------	----------------	----------------	----------------	----------------	----------------	----------------	----------------	----------------	----------------	----------------	----------------	----------------	----------------	----------------	----------------	----------------	----------------	----------------	----------------	----------------	----------------	----------------	----------------	----------------	----------------	----------------	----------------	----------------	----------------	----------------	----------------	----------------	----------------	----------------	----------------	----------------	----------------	----------------	----------------	----------------	----------------	----------------	----------------	----------------	----------------	----------------	----------------	----------------	----------------	----------------	----------------	----------------	----------------	----------------	----------------	----------------	----------------	----------------	----------------	----------------	----------------	----------------	----------------	----------------	----------------	----------------	----------------	----------------	----------------	----------------	----------------	----------------	----------------	----------------	----------------	----------------	----------------	----------------	----------------	----------------	----------------	----------------	----------------	----------------	----------------	----------------	----------------	----------------	----------------	----------------	----------------	----------------	----------------	----------------	----------------	----------------	----------------	----------------	----------------	----------------	----------------	----------------	----------------	----------------	----------------	----------------	----------------	----------------	----------------	----------------	----------------	----------------	----------------	----------------	----------------	----------------	----------------	----------------	----------------	----------------	----------------	----------------	----------------	----------------	----------------	----------------	----------------	----------------	----------------	----------------	----------------	----------------	----------------	----------------	----------------	----------------	----------------	----------------	----------------	----------------	----------------	----------------	----------------	----------------	----------------	----------------	----------------	----------------	----------------	----------------	----------------	----------------	----------------	----------------	----------------	----------------	----------------	----------------	----------------	----------------	----------------	----------------	----------------	----------------	----------------	----------------	----------------	----------------	----------------	----------------	----------------	----------------	----------------	----------------	----------------	----------------	----------------	----------------	----------------	----------------	----------------	----------------	----------------	----------------	----------------	----------------	----------------	----------------	----------------	----------------	----------------	----------------	----------------	----------------	----------------	----------------	----------------	----------------	----------------	----------------	----------------	----------------	----------------	----------------	----------------	----------------	----------------	----------------	----------------	----------------	----------------	----------------	----------------	----------------	----------------	----------------	----------------	----------------	----------------	----------------	----------------	----------------	----------------	----------------	----------------	----------------	----------------	----------------	----------------	----------------	----------------	----------------	----------------	----------------	----------------	----------------	----------------	----------------	----------------	----------------	----------------	----------------	----------------	----------------	----------------	----------------	----------------	----------------	----------------	----------------	----------------	----------------	----------------	----------------	----------------	----------------	----------------	----------------	----------------	----------------	----------------	----------------	----------------	----------------	----------------	----------------	----------------	----------------	----------------	----------------	----------------	----------------	----------------	----------------	----------------	----------------	----------------	----------------	----------------	----------------	----------------	----------------	----------------	----------------	----------------	----------------	----------------	----------------	----------------	----------------	----------------	----------------	----------------	----------------	----------------	----------------	----------------	----------------	----------------	----------------	----------------	----------------	----------------	----------------	----------------	----------------	----------------	----------------	----------------	----------------	----------------	----------------	----------------	----------------	----------------	----------------	----------------	----------------	----------------	----------------	----------------	----------------	----------------	----------------	----------------	----------------	----------------	----------------	----------------	----------------	----------------	----------------	----------------	----------------	----------------	----------------	----------------	----------------	----------------	----------------	----------------	----------------	----------------	----------------	----------------	----------------	----------------	----------------	----------------	----------------	----------------	----------------	----------------	----------------	----------------	----------------	----------------	----------------	----------------	----------------	----------------	----------------	----------------	----------------	----------------	----------------	----------------	----------------	----------------	----------------	----------------	----------------	----------------	----------------	----------------	----------------	----------------	----------------	----------------	----------------	----------------	----------------	----------------	----------------	----------------	----------------	----------------	----------------	----------------	----------------	----------------	----------------	----------------	----------------	----------------	----------------	----------------	----------------	----------------	----------------	----------------	----------------	----------------	----------------	----------------	----------------	----------------	----------------	----------------	----------------	----------------	----------------	----------------	----------------	----------------	----------------	----------------	----------------	----------------	----------------	----------------	----------------	----------------	----------------	----------------	----------------	----------------	----------------	----------------	----------------	----------------	----------------	----------------	----------------	----------------	----------------	----------------	----------------	----------------	----------------	----------------	----------------	----------------	----------------	----------------	----------------	----------------	----------------	----------------	----------------	----------------	----------------	----------------	----------------	----------------	----------------	----------------	----------------	----------------	----------------	----------------	--