Frieze Groups

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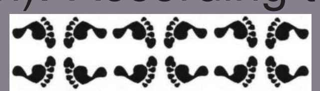
Introduction

Our chosen topic is frieze groups. Frieze groups are two-dimensional line groups, having repetition in only one direction. They are the distancing preserving transformations of a pattern.

What makes a frieze group

There are seven distinct frieze groups. All of them can be generated by translation, reflection (along the same axis) and a 180° rotation.
The seven Frieze groups are:

- The first frieze group F_1 was named by Conway as a HOP. 
- The second frieze group, F_2, contains translation and glide reflection symmetries. According to Conway, F_2 is called a STEP. 
- The third frieze group, F_3, contains translation and vertical reflection symmetries. Conway named F_3 a SIDLE. 
- The fourth frieze group, F_4, contains translation and rotation (by a half-turn) symmetries. According to Conway, F_4 is called a SPINNING HOP. 
- The fifth frieze group, F_5, contains translation, glide reflection and rotation (by a half-turn) symmetries. Conway calls F_5 a SPINNING SIDLE. 
- The sixth frieze group, F_6, contains translation and horizontal reflection symmetries. Conway named F_6 a JUMP. 
- Finally, the seventh frieze group, F_7, contains all symmetries (translation, horizontal vertical reflection, and rotation). According to Conway, F_7 is named a SPINNING JUMP. 

[1]

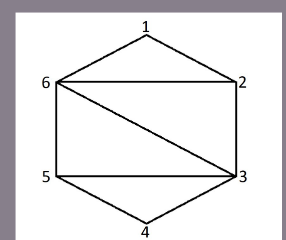
Background and History of frieze groups

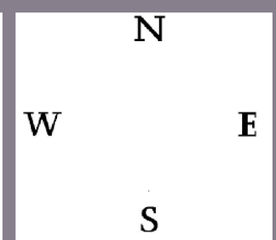
Frieze patterns name originated from the architectural term of a frieze or a broad decorative band and were extremely popular in Ancient Greece. The patterns started off as simply patterns of lines repeated all the way around the building, with each set of lines spaced a particular distance away from the previous one. Later on the patterns became more intricate involving moldings or painting in each of the spaces where the lines used to be, but it would still be the same image repeated all the way around the structure. [2]

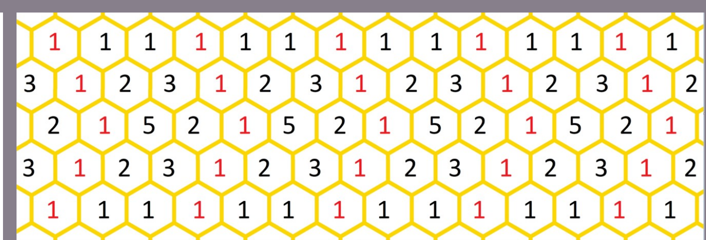
Frieze Pattern

Imagine some $n-gon$. Another way of looking at Frieze pattern is as a table of Natural numbers displayed in a lattice. Where the top and bottom rows are 1's, and the amount of rows is determined by $n-3$. To figure out the second row we triangulate the $n-gon$(hexagon in this example) any way we wish. Making some order out of the vertices(clockwise in this example), the number in the pattern corresponds to the amount of triangles adjacent to that vertex . The following rows are calculated by making unit diamonds with the above two rows labeling vertices in a compass fashion N,S,E,W.

$$(W \times E) - (N \times S) = 1$$



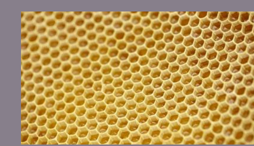
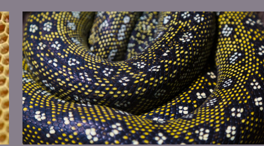




The example portrayed here is a special *Lightning bolt* example named due to the 'lightning bolt' of 1's

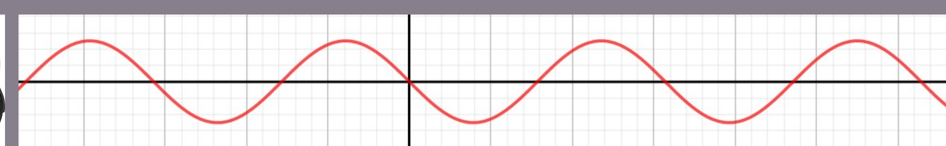
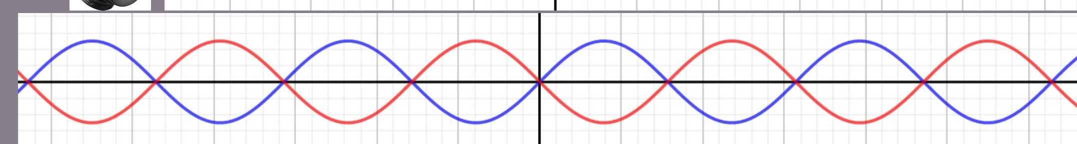
Examples of frieze groups in the real word

There are natural and man made friezes all around us. The honeycomb in a bees' nest (left) is an example of F_1 layered on top of each other. Snake skins have amazing intricate frieze patterns. This snake skin pattern (right) contains all symmetries. What Frieze groups can you spot in the honeycomb?

If we can think of a constant sound, like the engine on a plane, wave we can see examples of different frieze groups. If we take our first space as one period of a wave we can then see an example of F_1. If we take our second space as a quarter period of a wave we can then see an example of F_5.


An example of where we see similarities to Frieze groups are destructive sound waves, these are waves generated by headphones to cancel out background noise. Destructive waves looks like F_6 applied to the sound wave resulting in the inverted shape to cancel out noise(red graph). The resulting waves when added together(played together) should look like the final graph(red and blue) called total destructive interference.

[3]

References

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