History of Lagranges Theorem

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Introduction

Joseph Louis Lagrange and his theorem are core principals to grouping theory. Through this informative poster we intend to examine the history of this theorem including:

- A brief history on the creator Joseph Louis Lagrange
- Why did he create this theorem?
- Other famous mathematicians' contributions to the theorem
- Examples of this theorem
- Applications of the theorem in today's world

Joseph Louis Lagrange - The Man Behind The Theorem

Joseph Louis Lagrange was born on the 25th of january 1736 in Turin. He studied at the University of Turin where his favourite subject was classical Latin having no great enthusiasm in mathematics and found Greek Geometry rather dull and in his later life is famously quoted as saying "If I had been rich, I probably would not have devoted myself to mathematics".[1] After having his interest in mathematics sparked by reading a paper published by Edmond Halley. From this he improved his stature in the world of mathematics with feats like solving the isoperimetrical problem at only 19 years of age. Then in 1766 he moved to Berlin to start work on his theorem of which is named after him, Lagrange's Theorem. Later in life he moved to France and became a naturalised frenchman. He would pass away in Paris in April 1813.

What was he trying to achieve?

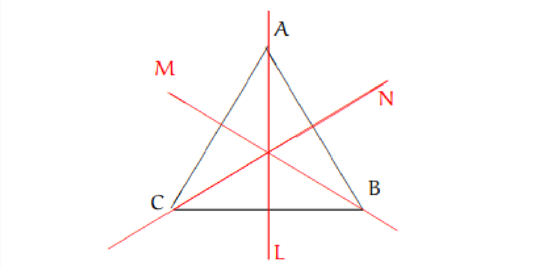
With formulas already existing for the quadratic, cube and quartic equations, Lagrange wanted to derive a formula for equations of degree five also known as quintic equations and more specifically for equations of nth degree. He saw that when solving more known polynomials such as quadratic and cubic resolvent polynomials of lesser degrees. From this he noted that the roots of the polynomial x1,x2,x3 and x4 could be permuted 24 times or (4!)

Other Famous Mathematician's Contributions

Augustin-Louis Cauchy contributed twice to Lagrange's theorem writing a paper on permutation groups before the idea of groups was even formalised and again in 1844, proving lagranges theorem for symmetric groups.
Camille Jordan added to Cauchy's work in 1861 by proving the theorem for finite permutation groups
Evariste Galois was the man who formalised the idea of groups in 1831 in a paper on solving solutions of permutation groups by radical

Examples of this Theorem

- Lagrange's Theorem: If G is a finite group and H is a subgroup of G, then the order of H divides the order of G.
- Let D6 be the set of symmetries of the equilateral triangle, with rotations id, R120, R240 and reflections TL, TM and TN as shown.[2]



- Then H = id, TL is a subgroup of D6 of order 2, and left cosets of H in D6 determined by the six elements are:

1. idH = id ∘ id, id ∘ TL = id, TL = H
2. TLH = TL ∘ id, TL ∘ TL = TL, id = H again.
3. R120H = R120 ∘ id, R120 ∘ TL = R120, TM.
4. TMH = TM ∘ id, TM ∘ TL = TM, R120 = R120H again.
5. R240H = R240 ∘ id, R240 ∘ TL = R240, TN
6. TNH = TN ∘ id, TN ∘ TL = TN, R240 = R240H again.

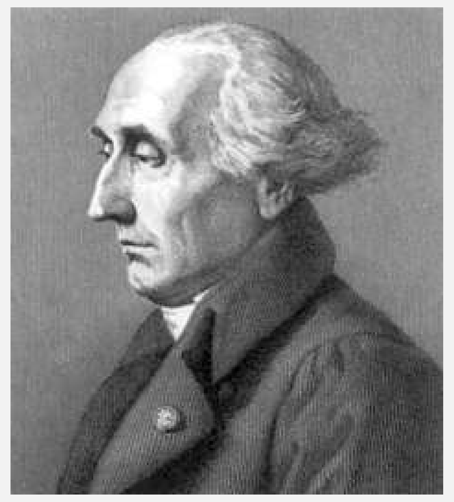


Figure: Joseph Louis Lagrange

Contributors To This Theorem

Figure: Cauchy(left), Galois(Centre), Jordan(right)

Application in Todays World

- It seems fair to state that Lagranges theorem is going to be used a lot more in the everyday life of a mathematician than that of say an electrician. If you are a mathematician however its applications are bountiful. For example, Lagranges theorem can be used to prove Euler's theorem as well as Fermat's little theorem aswell as its generalization.[3]
- It also has uses in cryptography.Due to its use in computing the power of an integer modulo a prime number. This shows the theorem to be applicable even in the general public's lives whether they know it or not as cyber-security plays such an important part in our lives.

Conclusion

Lagrange is such an interesting Mathematician to study to say the least. It seems bizzare that someone so intelligent and mathematically skilled had no great passion for his field of study. Inspite of this he has produced one of the most important theorem's used in Group theory to this day. It's core use in group theory will preserve Lagranges name in the minds of mathematicians for centuries to come.

References

[1] J. J. O'Connor and E. F. Robertson.
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`https://mathshistory.st-andrews.ac.uk/Biographies/Lagrange/`,, January 1999.

[2] Essential concepts of group theory.
`http://www.maths.nuigalway.ie/~rquinlan/groups/section2-1.pdf`.

[3] Lagranges theorem (group theory) wikipedia.
`https://en.wikipedia.org/wiki/Lagrange%27s_theorem_(group_theory)`.