# Group Theory Poster

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## Introduction

This is a poster about my group theory project. I investigated the topic of the original formulation of the groups concept for my project.

## What is group theory?

In mathematics and abstract algebra, group theory studies the algebraic structures called groups, which are systems consisting of a set of elements and a binary operation that can be applied to two elements of the set, which together satisfy certain axioms. A group must be closed under the binary operation, it has to follow the associative law, every element has to have an inverse, and there has to be an identity element. The concept of a group is central to abstract algebra. Other well-known algebraic structures, such as rings, fields, and vector spaces, can all be seen as groups endowed with extra operations and axioms. Groups appear again and again throughout mathematics, and the methods of group theory have influenced many parts of algebra. Linear algebraic groups and Lie groups are two parts of group theory that have both expanded to become subject areas in their own right.

## Mathematical predecessors

There were three main areas that led to the development of group theory:

- Early nineteenth Century geometry
- late eighteenth Century number theory
- the late eighteenth Century theory of algebraic equations which eventually led on to the study of permutations

## Geometry

At the beginning of the 19th Century geometry began to move on from the more elementary geometries of the past to the study of projective and non-euclidean geometries. The movement to study geometry in n dimensions also led to an abstraction within geometry itself. Monge, his student Carnot and Poncelet all contributed to the division between metric and incidence geometry.
Non-euclidean geometry was studied by Lambert, Gauss, Lobachevsky and János Bolyai among others.
Möbius in 1827, without any knowledge of the mathematical concept of a group, began to classify different geometries based on the fact that the properties that a particular geometry studies are invariant under a particular group.

## Number theory

Euler In 1761, studied the remainders of powers of a number modulo n. Although Euler's work isn't explicitly written in the language of group theory he does provide an example of the decomposition of an abelian group into cosets of a subgroup, and manages to prove a special case of the order of a subgroup being a divisor of the order of the group.
In 1801 Gauss expanded on Euler's work and contributed a lot to modular arithmetic which amounts to a fair amount of theory of abelian groups. He examined orders of elements and proved that there is a subgroup for every number dividing the order of a cyclic group, and he examined the behaviour of forms under transformations and substitutions. He partitioned forms into classes and then defined a composition on the classes. Gauss proved that the associative law held. In fact Gauss had a finite abelian group and in 1869, Schering, who edited Gauss's works, managed to find a basis for this abelian group.

## Theory of algebraic equations

Lagrange started out in 1770 with the study of permutations. Some noteworthy key players in the further development on Lagrange's work into the study of permutations and the development of group theory were Cauchy, Galois, and Cayley.

## Group theory outside mathematics

Group theory has made quite the visible impact outside the world of pure mathematics. It is quite common in string theory and particle physics. Particle physics used group theory to predict new particles arising from the symmetry that was seen in particles they already had knowledge of. All of the predictions turned out to be correct in the end. Group theory is also used to study the structure of crystals.

## References

https://mathshistory.st-andrews.ac.uk