School of Mathematics, Statistics and Applied Mathematics
Group Theory Poster Project
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Frieze Groups and Mass Housing

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Introduction

There exists many philosophies among architects as to what constitutes good architecture. From the artistic vision down to the budget which funds it, dozens of factors are influencing what a structure will look like. With the rise of software such as autoCAD, which allows the modern architect to manipulate 2d geometry to design 3d models, a portal has been opened between what can and cannot be done with respect to exploring new ways in approaching architectural projects. Mass housing, in particular, has created a unique opportunity to combine structural design with group theory in new and interesting ways. We intend to show how group theory, in particular the application of frieze groups, can be employed to relieve the monotony of mass housing in a way that distinguishes each unit from its neighbour while maintaining the fundamental architectural properties of symmetry. This new approach towards repetitive architecture revitalises the stale aesthetics of mass housing, with methods that are precise, efficient and innovative. The thesis of this project is largely built on the work of Jin-Ho Park, as well as Alice V. James, Davd A. James, Loukas N. Kalisperis and Cornelie Leopold.

Principal Objectives

1. Introduce the concept of frieze groups.
2. Discuss the seven types of frieze groups.
3. Relate frieze groups as a solution to the issue of mass housing.

1 Frieze Groups

In design, a frieze is a pattern that regularly repeats along a given direction. Often, these friezes appear horizontally along a wall or bench. In group theory, we imagine that friezes are infinite; extending left and right. All friezes are constructed such that, if they are moved to the left or right by one unit, the overall appearance of the frieze is left unchanged. Also, there may be ways of rotating or reflecting friezes that leave its appearance unchanged. However not all friezes have this property. Each frieze belongs to a frieze group. The elements of the associated frieze group are the actions that leave the frieze's appearance unchanged. There are 5 actions that can be performed on friezes: (1) Translations r, (2) Vertical Mirror M_v, (3) Horizontal Mirror M_h, (4) Half Turn 1/2, and (5) Glide Reflections g. Using these actions we can construct the 7 standard frieze groups.

Mirrors		
Type 1:	XXXXX	*Horizontal & vertical mirrors & half turns (m_h, m_v, 1/2)*
Type 2:	VVVVV	*Vertical mirrors & half turns & glide reflections (m_v, 1/2, g)*
Type 3:	AAAAA	*Vertical mirrors & no half turn (m_v)*
Type 4:	EEEEEE	*Horizontal mirrors & no half turn (m_h)*
No Mirrors		
Type 5:	SSSSSS	*Half turns (1/2)*
Type 6:	qdqdqdq	*Glide reflections & no half turns (g)*
Type 7:	RRRRR	*Translation only*

Table 1: The seven standard frieze types.

2 Frieze Patterns in Pirgí

In 2004, a paper[1] publised by Alice V. James, Davd A. James and Loukas N. Kalisperis explored the façades of friezes in the village of Pirgí on the Greek island of Chios. The paper discovered a series of patterns which "are used to create a lively geometry, ranging from the straightforward to the complex, to give each house its distinctive identity, its own unique face to display to the world. While analyzing the frieze designs, the authors discovered that the frieze artists intuitively obey a unique set of color-reversing rules."

2.1 Notes

The paper offers three important findings: *(1)* that the geometries used in Pirgí obeyed the laws of frieze theory; *(2)* that all seven frieze types were used across the decorations of the village (see Figures 1-7); *(3)* and that frieze groups offers a unique way to establish a sense of diversity that works best when the structures are uniform and even identical.

2.2 Examples

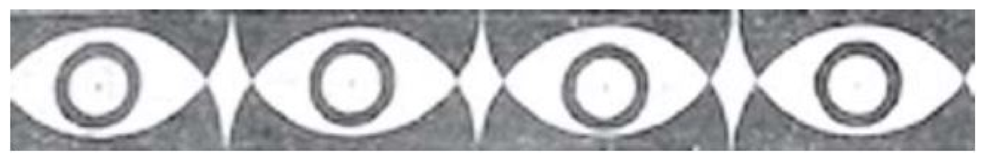

Figure 1: Type 1, Horizontal and vertical mirrors, half turns (m_h, m_v, 1/2)

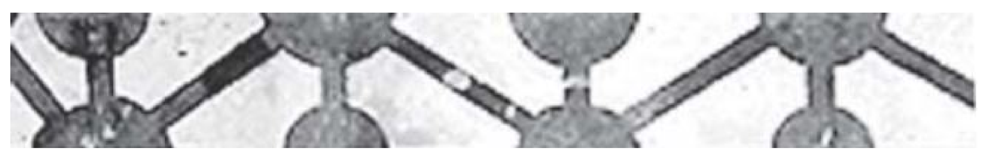

Figure 2: Type 2, Vertical mirrors, half turns, glide reflections (m_v, 1/2, g)

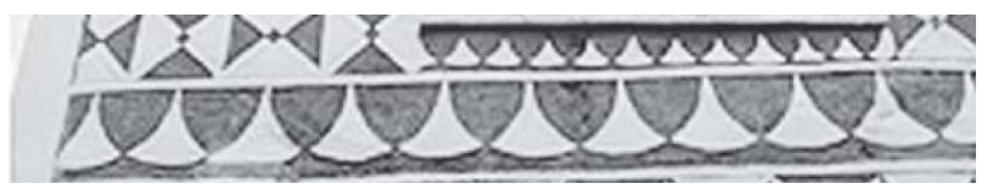

Figure 3: Type 3, Vertical mirrors and no half turns (bottom frieze) (m_v)

Figure 4: Type 4, Horizontal mirrors and no half turns (m_h)

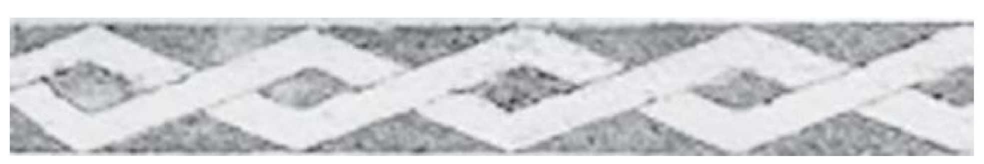

Figure 5: Type 5, Half-turns and no mirrors (1/2)

Figure 6: Type 6, No mirrors, glide reflections (g)

Figure 7: Type 7, Translation only, marching right triangles

3 Modern Solutions to Modern Problems

Evidently, if you were to hold these examples up to a mathematical lens, you would undoubtedly find that these patterns are not exactly obeying the laws of group theory; this is primarily due to the fact that these façades were hand-painted and likely did not use strict measurements. However, an exact rendition of these patterns has been made infinitely easier since the conception of applications like autoCAD and ProE.

3.1 Mass Housing

As hinted in the introduction, the practice of architecture is subjective, in this project we consider the design of mass housing as the primary subject since it has more limitations (e.g. government funded, generally attached/semi-detached, relatively small units) and the crucial property of being repetitive. The fundamental application of frieze groups will be in making the façade of housing less repetitive by combining common elements in housing to subvert the generic, dissatisfying, monotonous traits typically associated with it in a cheap and efficient way.

3.2 The Work of Jin-Ho Park

In 2017, Jin-Ho Park published a paper, entitled "Subsymmetries for the Analysis and Design of Housing façades[2]," in which he sets forth an innovative approach in applying frieze groups (and combinations of their subsymmetries) to repetitive rows of housing. The application process involved using computer software to generate a 3-dimensional model of the streetscape. As mentioned already, not only does Park employ the concepts of freize theory but also introduces a "combinatorial" approach in which he combines different constituents of symmetry and subsymmetries. In his own words, "by using all or just some of the subsymmetry principles, the application results in a huge number of compositional possibilities."

3.3 Park's method

1. A series of 2-dimensional diagrams are created with respect to the frieze groups[3]. These are unique elements of the façade (not the fundamental building structure) such as doors, partial walls, windows, etc., as seen in Table 2 below:

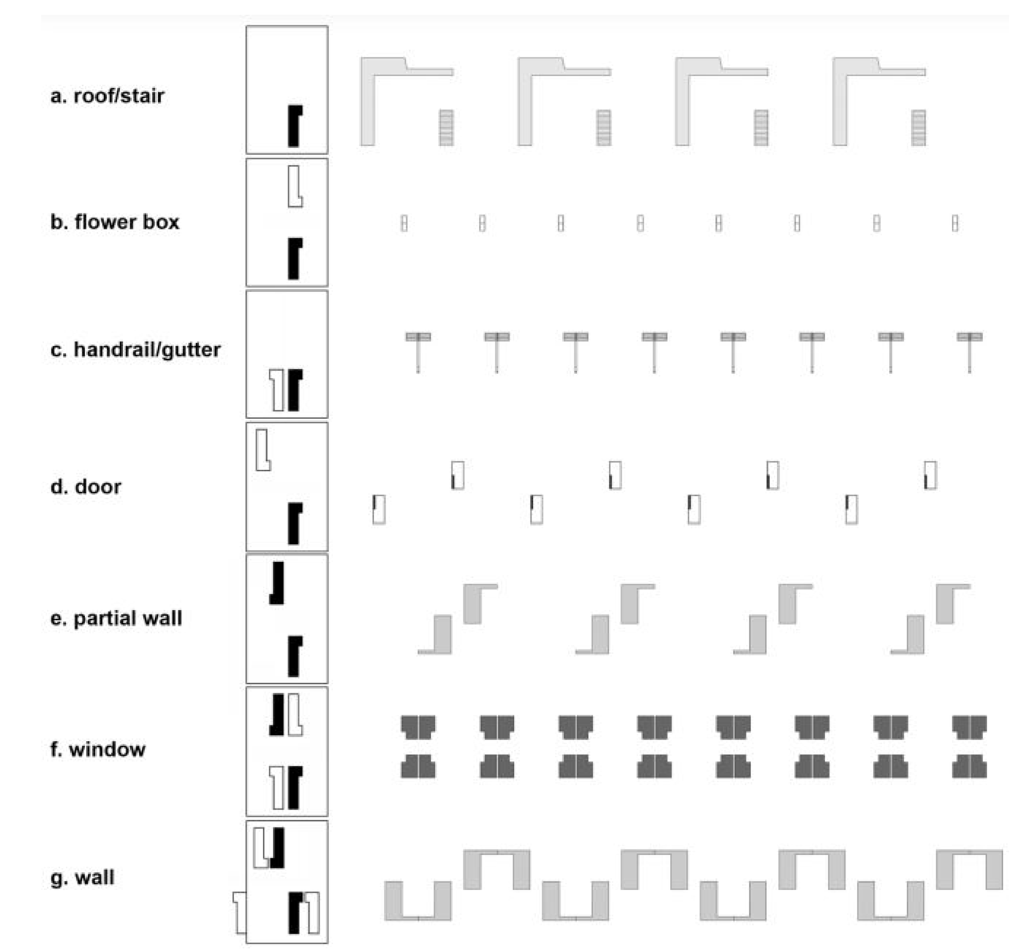

Table 2: Seven frieze groups with design elements: a detailed example of how the complete set of frieze groups is used to design the façade

2. These elements are then ***uniformly*** superimposed on each other without any variation in subsymmetries in Figure 8:

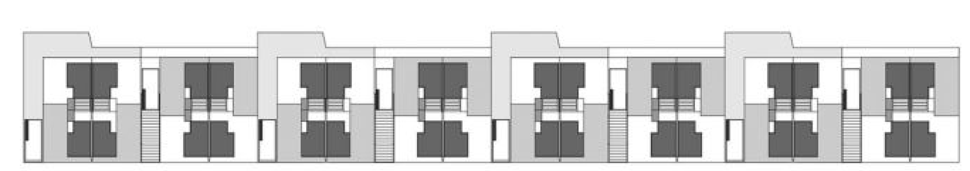

Figure 8: Final design where all elements are superimposed to form a housing façade

This creates a typical uniform housing façade. As Park notes, "Proper positioning of each symmetric element produces an orderly superimposed pattern of seven distinct symmetry operations. When overlaid together, separate building elements dissolve in a façade, having no meaning as separate rules of their juxtaposition in the entire façade, and making each underlying layer invisible."

3. Park then renders the façade elements in 3-d space, in which the underlying geometry is "not revealed," or subdued, due to its inherent asymmetrical design - offering a "dynamic look and aesthetic variety" regardless of the underlying uniform layout of subsymmetries.

Figure 9: Three-dimensional computer model is depicted on a streetscape

4. Finally, Park begins to vary the subsymmetries of the façade: "Depending on how different parts of elements are superimposed on the façade, the expression of the final design will be different, even though it may look as if various elements are permutated, shifted, and positioned. When combined with different colors and materials, dynamic views of the alternating façades are created... In addition, a few elements may be removed, or a few subsymmetry principles left unused, thereby destroying the overall symmetry of the façade." Park presents four unique iterations[4] of the same underlying principle in Figure 10:

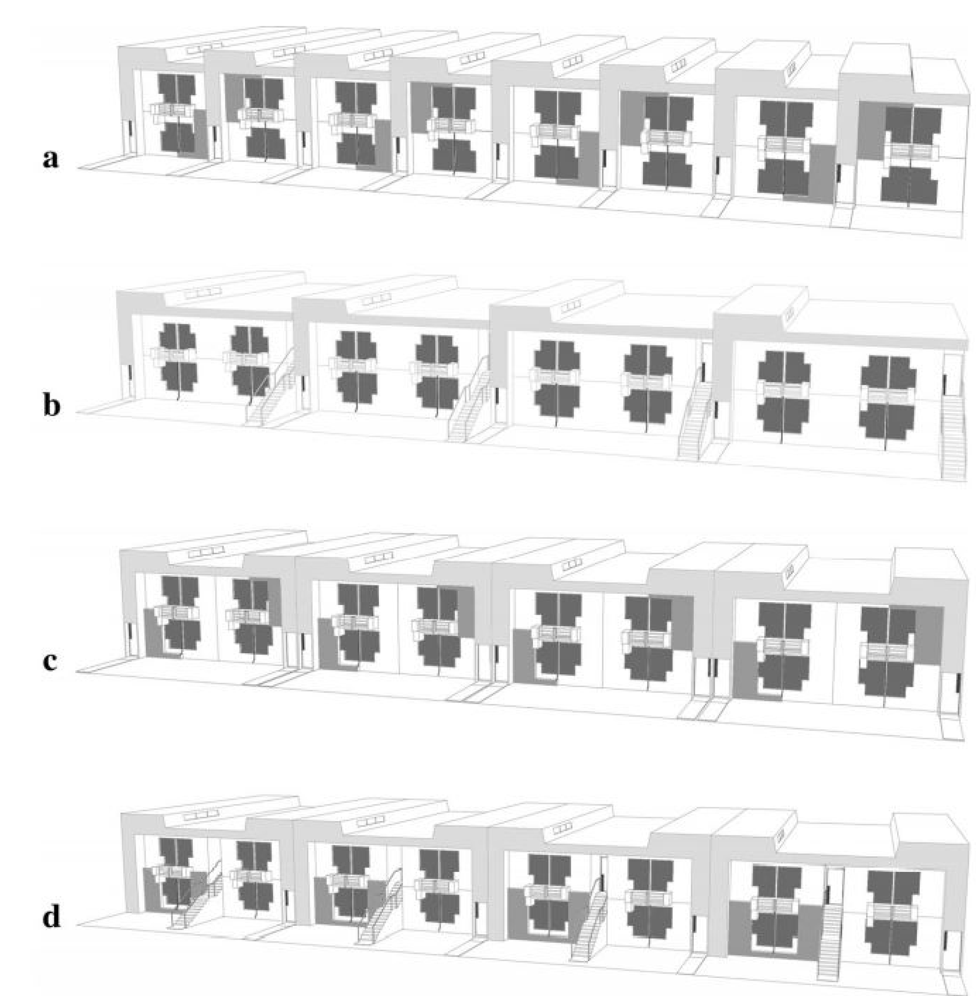

Figure 10: Four possible façade designs in that a few elements are removed or a few subsymmetry principles are not used. Although they appear similar, they are different from the ways that the principles are applied

4 Conclusion

- Group theory, and in particular the concept of frieze groups, has huge importance in structural and façade design.
- Although it has manifested in the past, modern technology now allows frieze patterns to be rendered with precision in complex and innovative ways.
- The first technological procedure for the technical application of frieze groups in architectural design was proposed (very recently) by Jin-Ho Park, whose method we have summarised and explored.
- This method can be universally applied to resolve the monotony of mass housing in a cheap, efficient way.

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References

[1] A. James, D. James, L. Kalisperis, A Unique Art Form: The Friezes of Pirgi (LEONARDO, Vol. 37, No. 3, 2004) p. 235.

[2] J. Park, Subsymmetries for the Analysis and Design of Housing Facades (Nexus Network Journal, 2017).

[3] *"In Fig. 9a, the pattern is shown rooted at the successive translation of asymmetric motifs by a distance: a wall and a stair. Figure 9b portrays a pattern of flower boxes. This example illustrates where a window is translated in the line of axis and then mirrored to generate the pattern. Paired, with corner elements, windows establish a rhythm across the units. In Fig. 9c, a gutter and a lanai handrail are mirrored with a subsequent translation. Figure 9d isolates the door and aligns it along a glide reflection. Here again, a decorative lighting element is attached to remove the symmetry of the door rectangle. The motifs in Fig. 9e are paired in a half turn. This forms a partial wall boundary of a unit of row houses. Figure 9f presents a window motif in Fig. 9b that is mirrored in a half-turn and reflected in an axis line. In Fig. 9g, a wall is half-turned and reflected in two mirrors at right angles. This forms the boundary configuration of all eight row house units. All the above generates a unique pattern in forming a fac¸ade of rows of houses."* J. Park, Subsymmetries for the Analysis and Design of Housing Facades (Nexus Network Journal, 2017).

[4] *"The first model of these (Fig. 12a) is a row of houses, where each unit has a clerestory window. The exterior stair is removed and the front door for each row house is placed on the ground level. The pattern for the slightly projecting wall is also changed. In this model, the Pma2 and P1a1 subsymmetries of the frieze groups are removed. In the second model (Fig. 12b), the projecting wall pattern is removed and the exterior stair is relocated. Two units are stacked together so that the two units appear to be a single building. In this model, the Pma2 and P112 subsymmetries of the frieze groups are removed. In the third model (Fig. 12c) each unit has its own roof but the forms reflect each other horizontally. The front door is relocated and the exterior stair is removed. In this model, the P1a1 and P111 subsymmetries of the frieze groups are removed. The fourth model (Fig. 12d) removes the projecting wall of the upper floor. The roof form is reflected and the exterior stair is translated. In this model, the Pma2, P112, and P111 subsymmetries of the frieze groups are removed."* J. Park, Subsymmetries for the Analysis and Design of Housing Facades (Nexus Network Journal, 2017).

[5] C. Leopold, Geometry Concepts in Architectural Design (ReseachGate).

[6] Wikipedia (2020) Group Theory, available: *https : //en.wikipedia.org/wiki/Group_theory* (accessed 18 November 2020).