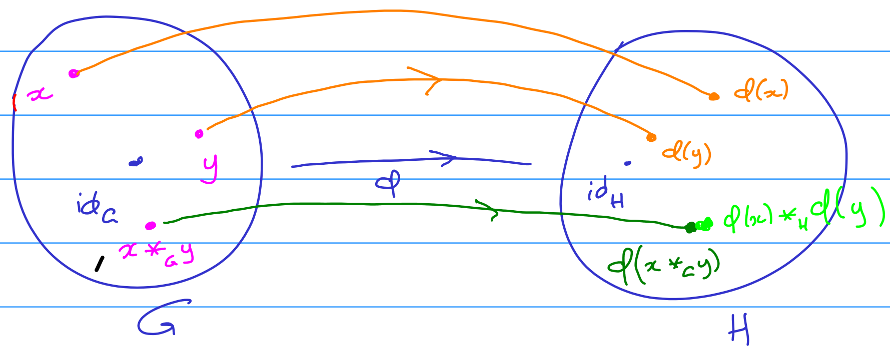

$\phi : G \longrightarrow H$ a function

ϕ is a group homomorphism $\quad \phi(x *_G y) = \phi(x) *_H \phi(y)$

for all $x, y \in G$

Example $\quad \det : GL(2,\mathbb{R}) \longrightarrow \mathbb{R}^\times \longleftarrow$ non-zero real numbers under multiplication

Let $A, B \in GL(2,\mathbb{R})$

$$\det(A \cdot B) = \det(A) \times \det(B)$$

matrix product $\qquad$ product in $\mathbb{R}$

$GL(2,\mathbb{R}) \longrightarrow GL(2,\mathbb{R}) \qquad A \longrightarrow A^{-1}$

Take $A, B \in GL(2,\mathbb{R})$

$$\boxed{(AB)^{-1} \overset{?}{=} A^{-1} B^{-1}} \quad \times$$

not a homomorphism

In fact $(AB)^{-1} = B^{-1}A^{-1}$, which is generally not equal to $A^{-1}B^{-1}$ in $GL(2,\mathbb{R})$

$\mathbb{R}^\times$

④ $(\mathbb{Z}, +) \to (\mathbb{R}\setminus\{0\}, \times)$ $\qquad x \to 2^x$

Let $a, b \in \mathbb{Z}$

$$2^{a+b} = 2^a \times 2^b \quad \checkmark \quad \text{Yes}$$

⑤ $GL(2,\mathbb{R}) \to (\mathbb{R}, +)$

$A \to A_{11}$ $\qquad$ e.g. $\begin{pmatrix} 2 & 1 \\ 1 & 4 \end{pmatrix} \to 2$

If $A, B \in GL(2,\mathbb{R})$ $\qquad (AB)_{11} \overset{?}{=} A_{11} + B_{11}$ $\qquad$ No

$\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$ $\qquad (AB)_{11} = 1 \neq A_{11} + B_{11} = 2$

e.g. $\quad A = B = I_2$ $\quad$ then $\quad AB = I_2$ also