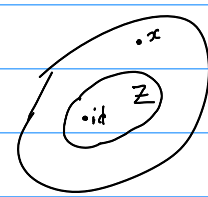
Q3 Homework 2

For every group G, $\overset{Z}{Z(G)}$ is a <u>normal</u> subgroup of G.

To show this, we must show that for every element $\boxed{x}$ of G, the left coset xZ and the right coset Zx are the same set



left coset $\quad xZ = \{ xz : z \in Z \}$

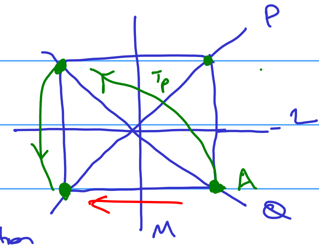
For every $z \in Z$, we have $xz = zx$

So $xZ = \{ zx : z \in Z \} = Zx$ ← right coset of Z determined by x.

So $xZ = Zx$ for all $x \in G$

Q2 (Variant)

In D_8, look at the subgroup $K = \{id, T_L\}$



reflection in L

Claim $K \ntrianglelefteq D_8$

[$\trianglelefteq$: normal
$\ntrianglelefteq$: not normal]

Claim $T_p K \neq K T_p$ (right coset)

(left coset of K determined by T_p)

$T_p \circ T_L = R_{90}$

$$T_p K = \{T_p \circ \underline{id},\ T_p \circ \underline{T_L}\} = \{T_p, R_{90}\}$$

$$K T_p = \{id \circ T_p,\ \underbrace{T_L \circ T_p}_{R_{270}}\} = \{T_p, R_{270}\}$$

$T_p K \neq K T_p$

$\Rightarrow$ K is not a normal subgroup of D_8