Q12 How many elements of S_{11} have order 15?

An element has order 15 if the least common multiple of the lengths of its disjoint cycles is 15.

<u>Variant</u> No. of elements of S_{12} of order 15?

Cycle types - can't have a 15-cycle (only 12 elements)

- can have 5-cycles, and 3-cycles

[every cycle length must be 1, 3 or 5]
with at least one 3 and at least one 5]

Could have in S_{12}

2 classes to count

① (_ _ _ _ _) (_ _ _) (4 fixed points)

② (_ _ _ _ _) (1 2 3) (4 5 6) (one fixed point)

① $\binom{12}{5}$ choices for the entries in the 5-cycle, 4! ways to arrange them in cyclic order

$\binom{7}{3}$ choices for 3 remaining entries in the 3-cycle, 2! ways to put them in cyclic order

① $\binom{12}{5} \times 4! \times \binom{7}{3} \times 2$ ← elements in Class 1

② $\binom{12}{5} \times 4!$ choices for the 5-cycle (7 elements left)

$\binom{7}{3} \times 2$ choices for the 1st 3-cycle (4 elements left)

$\binom{4}{3} \times 2$ choices for the 2nd 3-cycle

Note: $\binom{12}{5} \times 4! \times \binom{7}{3} \times 2 \times \binom{4}{3} \times 2$ overcounts by a factor of 2 because it counts every combination of two 3-cycles twice e.g. $(1\ 2\ 3)(4\ 5\ 6) = (4\ 5\ 6)(1\ 2\ 3)$

- Divide by 2 to correct for this.

② $\binom{12}{5} \times 4! \times \binom{7}{3} \times \binom{4}{3} \times 2$

Total number $\left[\binom{12}{5} \times 4! \times \binom{7}{3} \times 2\right] \times \binom{?}{5}$

$\boxed{(1\ 2)(3\ 4)}$

Group action of a group G on a set S - element of G permute the element of S.

Orbit - Stabilizer Theorem

For $x \in S$, $G \cdot x = \{ g \cdot x \mid g \in G \} \subseteq S$

$\boxed{\text{orbit of } x}$

Stabilizer of x $\quad Stab_G(x) = \{ g \in G : g \cdot x = x \}$, a subgroup of G

$$|G \cdot x| = [G : Stab_G(x)]$$

Consequence This means that if G is finite, the number of elements in $G \cdot x$ is a divisor of $|G|$

Recall The set S separated into a collection of disjoint orbits by the action of G.