

Challenge 2

Write

$$S = \left\{ \begin{pmatrix} a & b \\ c & d \end{pmatrix} : a, b, c, d \in \mathbb{Z}, ad - bc \neq 0 \right\}.$$

So S is the set of 2×2 matrices with integer entries and non-zero determinant.

Does every element of S have an inverse in S ?

How can we quickly decide whether the inverse of a particular matrix in S is itself in S ?

For example, what about $\begin{pmatrix} 3 & 2 \\ 8 & 5 \end{pmatrix}$?