MA3343 Groups

Introduction - Lecture 1

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Group Theory and Abstract Algebra

Algebra is the study of algebraic structures. An algebraic structure is a set of objects (e.g. numbers, functions, matrices) with a binary operation.

A binary operation is a way of combining any pair of the objects in the set to produce a new object in the same set (e.g. addition of integers, multiplication of 2×2 matrices, composition of functions from $\mathbb{R}$ to $\mathbb{R}$).

It's possible to study the properties of binary operations on sets abstractly, without particular attention to the elements to which they apply. Different combinations of properties determine different types of algebraic structures, like vector spaces, groups, rings, fields and many others.

Planned structure of this module

Resources are at `maths.nuigalway.ie/ rquinlan/groups`

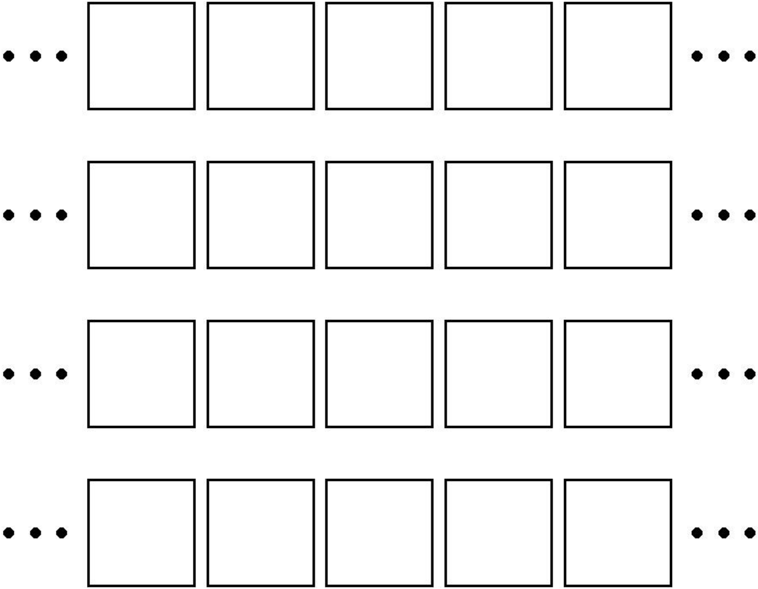
- Monday/Tuesday - email to all students with updates and plan for the week
- Thursday/Friday - lectures in person on campus (we will insist on face coverings and open windows)
- Tutorials (in person and/or online) will begin in Week 3 or 4. There we will
  - discuss current content;
  - look at homework problems;
  - work on poster projects;
  - learn how to use LaTeX with Overleaf.

Assessment - flexible with several elements

1. Weekly challenges - every week we will have a challenge, to create some content for the class in the form of a single slide or page. Successfully completed challenges are worth 2% each (total $\sim$20%).
2. Two homework assignments, in Week 3 and Week 6 approx, worth 12.5% each.
3. Group poster project - team up with two classmates and create a poster on a topic in group theory for our end of year exhibition in December (25%).
4. Final exam in the December exam session (up to 70%).

This adds up to more than 100%. You don't have to do all the components. You can choose according to your interests.

A bit more on the frieze groups



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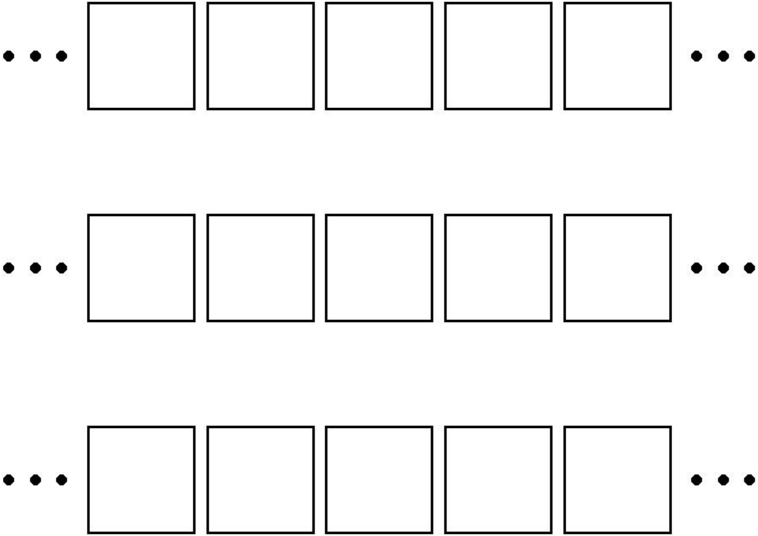

Challenge 1

Create a slide with your own pictures of friezes corresponding to the seven frieze groups. Make them look as different as possible from mine!

The order in which we have listed the seven groups are

1. Translations only;
2. Translations and 180° rotations only;
3. Translations and horizontal reflections only;
4. Translations and vertical reflections only;
5. Translations, horizontal and vertical reflections, and 180° rotations;
6. Translations, vertical reflections, 180° rotations and glide reflections only;
7. Translations and glide reflections only.

Note on submission and marking of the weekly challenges

Submission will be via Blackboard assignments. Each will be scored out of 10, according to the following scheme.

- 10 for a submission that fully answers the question and has an exceptonally high quality of exposition and explanation.
- 7 for a submission that answers the question (almost) fully or almost fully but is less than perfectly explained.
- Up to 5 for a submission that contains some relevant ideas but does not give a full explanation.

IMPORTANT: Identical submissions will get zero or very low marks. Working together is encouraged, but make a joint submission (up to three named authors) in that case. Submissions transcribed verbatim from elsewhere will get zero or very low marks. The weekly challenges are about your own creative expression - searching for relevant texts is encouraged, but giving your own explanation is part of the task.

Secton 1.1 Examples of Groups. Here is Example 1

$(\mathbb{Z}, +)$

Example 2

$(\mathbb{C}^\times, \times)$

Here $\mathbb{C}^\times$ denotes the set of *non-zero* complex numbers, and "$\times$" denotes multiplication of complex numbers.

Example 3

$(\mathrm{GL}(2, \mathbb{Q}), \times)$

Read this as "the general linear group of 2 by 2 matrices over the rational numbers" ("GL" stands for "general linear"). This time, $\times$ denotes matrix multiplication.

Example 4

$$(\{1, i, -i, -1\}, \times)$$

Example 5

Let S_3 denote the following set of 3×3 matrices.

$$S_3 = \left\{ \begin{array}{ccc} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}, & \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}, & \begin{pmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{pmatrix} \\ \\ \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix}, & \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{pmatrix}, & \begin{pmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{pmatrix} \end{array} \right\}.$$

What happens when you mutiply two elements of S_3? Do you get an element of S_3 (in algebra language, is this set S_3 *closed under matrix multiplication*)? If so, is this an accident, or does it follow from some special property of the matrices in S_3? Does S_3 have a neutral element for multiplication? Does every element of S_3 have an inverse in S_3 for multiplication?