

Section 2.2 The Centre, Centralizers and Conjugacy

Definition 2.2.1 Let (G, \star) be a group. The **centre** of G , denoted $Z(G)$, is the set consisting of those elements of G that commute with every element of G under the group operation.

$$Z(G) = \{x \in G : x \star g = g \star x \text{ for all } g \in G\}.$$

Examples

- ▶ The centre of $GL(3, \mathbb{R})$ is the set of scalar multiples of the identity matrix.
- ▶ The centre of D_8 is $\{\text{id}, R_{180}\}$.
- ▶ The centre of D_6 is the trivial subgroup.
- ▶ The centre of $(\mathbb{Z}, +)$ is the entire group.

The centralizer of an element

Definition 2.2.2 Let g be an element of a group (G, \star) . The **centralizer** of g in G , denoted $C_G(g)$, is the set of all elements of G that commute with g .

$$C_G(g) = \{x \in G : g \star x = x \star g\}.$$

Examples

- ▶ The centralizer in D_6 of R_{120} consists of the three rotations.
- ▶ The centralizer in D_6 of any one of the reflections consists only of that reflection and the identity element.