Section 2.1 Lagrange's Theorem

Theorem 2.1.1 (Lagrange's Theorem) Let G be a finite group and let H be a subgroup of G. Then the order of H divides the order of G.

Definition 2.1.2 (coset) Let G be a group with a subgroup H, and let $g \in G$. Then the *left coset of H in G determined by g* is the set

$$gH = \{gh : h \in H\}.$$

Then gH is a subset of G, and $gH = H$ if and only if $g \in H$.

Three steps to prove Lagrange's Theorem

1. Lemma 2.1.3 Suppose that H is a finite subgroup of a group G, and let $g \in G$. Then the left coset gH has the same number of elements as H.
2. Lemma 2.1.4 (this is the big one) Suppose that g_1 and g_2 are elements of a group G and that H is a subgroup of G. Then the left cosets g_1H and g_2H are either equal or disjoint - in other words they either coincide fully or don't intersect at all.
3. Lemma 2.1.5 The union of the distinct left cosets of H in G is all of G.