Normal Subgroups

A subgroup N of a group G is normal in G (written $N \trianglelefteq G$) if for every element g of G the left coset gN is equal to the right coset Ng.

Equivalently, N is normal in G if $g^{-1}ng$ belongs to N for every $n \in N$ and $g \in G$. This means that N contains all G-conjugates of all of its elements, or that N is a union of conjugacy classes of G.

Examples of Normal Subgroups

- Every group is a normal subgroup of itself, and the trivial subgroup is a normal subgroup of every group.
- Every subgroup of an abelian group is normal (conjugacy classes in abelian groups are single elements).
- Any subgroup that is the kernel of a group homomorphism is normal.
- In D_6 the subgroup consisting of the three rotations is normal, and the subgroup consisting of any one reflection is non-normal.
- Any subgroup of index 2 in any group is normal.