Group Homomorphisms

A function ϕ from a group G to a group H is a group homomorphism if

$$\phi(x \star_G y) = \phi(x) \star_H \phi(y), \ \forall \ x, y \in G.$$

The kernel of ϕ is the set of elements of G whose image under ϕ is the identity element in H.
The image of ϕ is the subset of H consisting of the images under ϕ of the elements of G.
Lemma 4.1.3 $\ker \phi$ is a subgroup of G.
Lemma 4.1.4 $\mathrm{Im}\phi$ is a subgroup of H.

Examples

1. The Determinant The kernel of the determinant homomorphism from $\mathrm{GL}(3, \mathbb{Q})$ to $\mathbb{Q}^\times$ is

$$\{A \in \mathrm{GL}(3, \mathbb{Q}) : \det A = 1\}.$$

   This group is called the special linear group, denoted $\mathrm{SL}(3, \mathbb{Q})$.

2. $\phi : \mathbb{Z} \to \{1, -1\}$ with

$$\phi(n) = \begin{cases} 1 & \text{for} \quad n \text{ even} \\ 2 & \text{for} \quad n \text{ odd} \end{cases}$$

   $\ker \phi$ is the set of even integers.