MA3343 Group Theory 2018-19
COURSE OUTLINE

LECTURER: Dr Rachel Quinlan
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LECTURES: Thursday 12.00 AC202, Friday 12.00 AC201
Tutorial schedule to be arranged in Week 3 or so.

OFFICE HOURS: Whenever the door is open

SYLLABUS

This course is an introduction to Group Theory. The subject is concerned with algebraic structures (sets) whose elements can be combined in pairs according to some operation (such as addition, multiplication, composition) in a manner that satisfies certain natural conditions known as the axioms of a group. Groups are defined in abstract terms but they are ubiquitous in mathematics, examples include the integers (with addition), the non-zero rational numbers (with multiplication), the permutations of a set (with composition), the 2×2 real matrices of non-zero determinant (with matrix multiplication), and so on. The idea of using the single word "group" to describe a diverse range of objects that share some properties in terms of their algebraic structure is a relatively newfangled one, but it has proved to be extremely powerful. Although abstract reasoning is a central and essential feature of group theory, we will have plenty of examples to give us some context. Many of our concrete examples will come from matrix algebra, which provides both a plentiful supply of different examples and a rich context for representing groups that arise in different situations.

Our syllabus will have four chapters.

1. What is a Group?
   Examples of groups, axioms of a group, subgroups and generating sets.
2. Essential concepts of group theory
   Abelian groups, the centre of a group, the centralizer and conjugacy class of an element. The order of a group and the order of an element. Lagrange's Theorem on the order of a subgroup of a finite group.
3. Group actions
   Groups acting on sets, with examples. The Orbit-Stabilizer Theorem. Isomorphism. Cayley's Theorem.
4. Group homomorphisms. Normal subgroups and quotient groups.

COURSE STRUCTURE

The lecture notes will be posted in instalments on the MA3343 Blackboard page as the course proceeds. They constitute the "text" for the course. Relevant sections of the notes will generally be available before the lecture takes place, and students are encouraged to download them in advance. The lecture time will be used to discuss the central themes and important questions and not to read the notes. There will be a series of three homework assignments as the course proceeds, one corresponding to each of the first three chapters of the lecture notes.

What students are expected to do

- Participate actively in lectures.
- Study the lecture notes on an ongoing basis. The lecture notes and the lectures are intended to complement each other, they are not alternatives to each other. Not everything in the notes will be discussed in lectures, and not everything that is discussed in the lectures will be detailed in the notes. In particular, the lecture time will not be used to read the notes. Everything in the lecture notes is included in the course, whether it is discussed in lectures or not (unless explicit advice to the contrary is given). It is a good idea to come to lectures with a copy of the relevant notes and to have had at least a superficial look at them in advance.
- Think carefully about the questions and exercises posed in the notes. A suggestion is to maintain your own more detailed version of the notes that includes your work on these items.
- Work hard on the homework problems. You may wish to team up with other students in the class in order to do this. By agreement with the lecturer it will be possible to submit one set of solutions from a group of up to three students.
- If you are experiencing difficulties with the course despite your reasonable efforts, bring this to the attention of the lecturer.
- Think about the learning outcomes below and work towards achieving them. Remember that the purpose of the exam is to determine, fairly and comprehensively, how well you have achieved the learning outcomes. Your best strategy for success in the exam is to concentrate on achieving the learning outcomes.

Learning Outcomes

By the end of this course you will be able to :

- Explain what a group is and use the definition of a group to identify examples and non-examples.
- Use the language and terminology of group theory in an accurate and knowledgeable way.
- Give examples of groups with certain specified properties, and determine whether a given group has a specified property.
- Explain the concept of a group action, and identify features of particular group actions.
- State and prove some significant theorems of group theory, including Lagrange's Theorem and the Orbit-Stabilizer Theorem.
- Critically assess proposed proofs of statements in group theory, and write some proofs of your own.

Assessment

The set of three homework assignments will account for 30% of the marks.
The two-hour final exam in the Winter exam session will account for 70% - more details on that later.

Supplementary Reading

Browse the library for books that introduce Group Theory/Abstract Algebra. Two examples are

- *A First Course in Abstract Algebra*, John B. Fraleigh (512.02)
- *Modern Algebra*, John R. Durbin (512.02 DUR)

Wikipedia is also a very good resource for information on group theory.

Useful background

Familiarity with matrix algebra - the mechanism of matrix multiplication and its meaning in terms of composition of linear transformations, and the concept of the inverse of a matrix. Familiarity with modular arithmetic (addition and multiplication modulo a positive integer) will be useful too.