Graph Theory (MA 522)

Problems to think about - Week 2

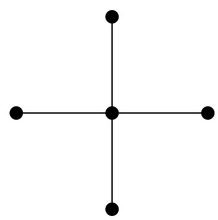
For discussion in Week 3 . . .

Note : Problems marked $\star$ are hard in my opinion. Problems marked with † are of some importance in the subject and should probably be discussed in next week's seminars. Exercises referenced are in Bondy and Murty's book.

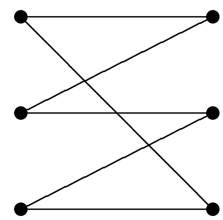
1. Give five examples of graphs whose automorphism group is trivial (i.e. consists only of the identity).

2. Describe the automorphism group of each of the following graphs.

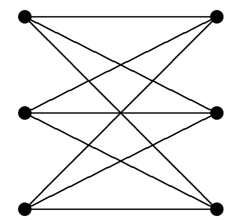
(a)



(b)



(c)



   (d) † The cycle graph C_n (n vertices arranged in a cycle).

   (e) † The *path* graph P_n (n vertices arranged in a path).

   (f) †$\star$ The complete bipartite graph $K_{m,n}$ (this is the bipartite graph with m vertices in one part, n in the other, and every vertex in the first part joined by an edge to every vertex in the second). Note : For (f), bear in mind that m may be equal to n.

3. $\star$ Give an example of a graph whose automorphism group is cyclic of order 3. Hint : There is no such graph on fewer than nine vertices.

4. Let G be a simple graph. Prove that G and G^c (the complement of G) have the same automorphism group.

5. †

   (a) Show that if the graph G is disconnected, then G^c is connected.

   (b) Is the converse true?

   Note : A proof that a graph is connected would be likely to start by saying something like "Let u and v be vertices in the graph. We need to be convinced that there is a path from u to v in the graph . . ."

6. † Show that if G is connected and simple but not complete, then G has three vertices u, v and w such that uv and vw are edges but uw is not an edge.