EGMO IRELAND

Practice Problems - February 5th 2012

Please post you comments on these problems here. You are encouraged to send written solutions to :
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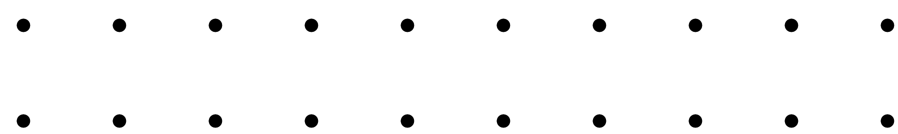
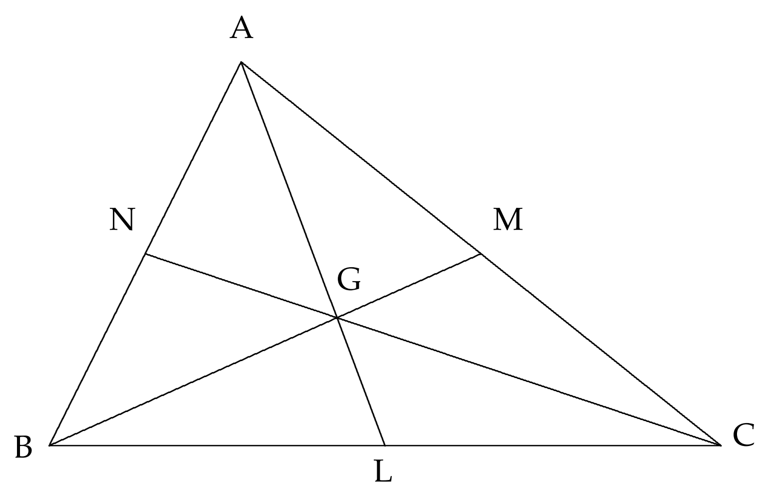
1. Two rows of 10 pegs each are lined up as in the diagram below. Within each row, adjacent pegs are 1 unit apart, and the parallel rows are also 1 unit apart.



   You have 10 rubber bands, each to be looped around a pair of pegs, in such a way that every peg is occupied by a rubber band. The maximum possible distance between a pair of pegs to which a rubber band can stretch is $\sqrt{2}$ units. In how many ways can you fit the rubber bands to the pegs?

2. In the triangle ABC below, L, M and N are the midpoints of the three sides. Prove that the triangles $\triangle ANG$, $\triangle BNG$, $\triangle BLG$, $\triangle CLG$, $\triangle MCG$ and $\triangle AMG$ all have the same area.



3. Suppose that n is an integer that is not a multiple of 2 or 5. Show that some multiple of n has all of its digits equal to 1.

4. Find all polynomials
$$f(x) = a_0 + a_1 x + a_2 x^2 + \ldots + a_n x^n$$
   satisfying the equation
$$f(x^2) = (f(x))^2$$
   for all real numbers x.

Comments overleaf

1. If it's not obvious what to do with this problem, simplify it - first to the case of two pegs (and one band), then four pegs, then six . . . see if you can figure out a pattern.

2. Note that the triangles $\triangle ABL$ and $\triangle ACL$ each account for half the total area. Same for $\triangle ACN$, $\triangle BCN$, $\triangle ABM$ and $\triangle BCM$. What can you do with this?

3. Look at the numbers $1, 11, 111, 1111, \ldots$ - can they all have different remainders on division by n? (This is not the answer but it is a step).

4. Two polynomials have the same values for every real number only if they are exactly the same - i.e. all their coefficients coincide.