1.3 The Reduced Row-Echelon Form (RREF)

Definition 1.3.1 *A matrix is in* reduced row-echelon form (RREF) *if*

1. *It is in row-echelon form, and*

2. *If a particular column contains a leading 1, then* all *other entries of that column are zeroes.*

If we have a row-echelon form, we can use ERO's to obtain a reduced row-echelon form (using ERO's to obtain a RREF is called *Gauss-Jordan elimination*).

Example 1.3.2 *In Example 1.2.4, we obtained the following row-echelon form :*

$$\begin{pmatrix} 1 & -1 & -1 & 2 & 0 \\ 0 & 1 & 4 & 3 & 10 \\ 0 & 0 & 1 & 1 & 2 \end{pmatrix} \qquad (\text{REF, not } \textit{reduced} \text{ REF})$$

To get a RREF from this REF :

1. Look for the leading 1 in Row 2 - it is in column 2. Eliminate the non-zero entry *above* this leading 1 by adding a suitable multiple of Row 2 to Row 1.

$$R1 \quad \to \quad R1 + R2 \qquad \begin{pmatrix} 1 & 0 & 3 & 5 & 10 \\ 0 & 1 & 4 & 3 & 10 \\ 0 & 0 & 1 & 1 & 2 \end{pmatrix}$$

2. Look for the leading 1 in Row 3 - it is in column 3. Eliminate the non-zero entries *above* this leading 1 by adding suitable multiples of Row 3 to Rows 1 and 2.

$$\begin{array}{ccc} R1 & \to & R1 - 3R3 \\ R2 & \to & R2 - 4R3 \end{array} \qquad \begin{pmatrix} 1 & 0 & 0 & 2 & 4 \\ 0 & 1 & 0 & -1 & 2 \\ 0 & 0 & 1 & 1 & 2 \end{pmatrix}$$

This matrix is in *reduced* row-echelon form.

The technique outlined in this example will work in general to obtain a RREF from a REF : you should practise with similar examples!

<u>Remark</u>: Different sequences of ERO's on a matrix can lead to different row-echelon forms. However, only one *reduced* row-echelon form can be found from any matrix.