OLLSCOIL NA hÉIREANN, GAILLIMH

NATIONAL UNIVERSITY OF IRELAND, GALWAY

AUTUMN EXAMINATIONS 2006

SECOND UNIVERSITY EXAMINATION

MATHEMATICS

MA 203 - *LINEAR ALGEBRA*

Dr D. Johnson
Professor J. Hinde
Professor T. Hurley
Dr R. Quinlan

Time Allowed : **Two** hours

Answer **three** questions.

---

1. (a) Find the general solution of the following system of linear equations.

$$\begin{array}{rcrcrcrcr} x_1 & - & 2x_2 & + & x_3 & - & 8x_4 & = & -6 \\ 2x_1 & + & x_2 & + & 2x_3 & - & x_4 & = & -2 \\ 2x_1 & - & 2x_2 & - & 3x_3 & + & 10x_4 & = & -3 \end{array}$$

(b) Find all solutions of the following system of linear equations.

$$\begin{array}{rcrcrcrcr} x_1 & - & 2x_2 & + & x_3 & - & 8x_4 & = & -6 \\ 2x_1 & + & x_2 & + & 2x_3 & - & x_4 & = & -2 \\ 2x_1 & - & 2x_2 & - & 3x_3 & + & 10x_4 & = & -3 \\ x_1 & - & 2x_2 & + & 2x_3 & - & x_4 & = & 15 \end{array}$$

(c) Show that the following system of linear equations is inconsistent.

$$\begin{array}{rcrcrcrcr} x_1 & - & 2x_2 & + & x_3 & - & 8x_4 & = & -6 \\ 2x_1 & + & x_2 & + & 2x_3 & - & x_4 & = & -2 \\ 2x_1 & - & 2x_2 & - & 3x_3 & + & 10x_4 & = & -3 \\ 2x_1 & - & 2x_2 & + & 2x_3 & - & 10x_4 & = & 1 \end{array}$$

P.T.O.

2. (a) State whether each of the following statements is true or false.

   (i) A system of linear equations having five equations and four variables can have a unique solution.

   (ii) If A and B are invertible square matrices of the same size, then $A - B$ must also be invertible.

   (iii) If there exists a consistent system of linear equations having a particular square matrix A as coefficient matrix, then *every* system of linear equations that has A as coefficient matrix is consistent.

   (iv) If A is a 3×3 matrix and $\det(A) = 1$, then $\det(2A) = 2$.

(b) Use Gauss-Jordan elimination to calculate the inverse of the matrix

$$\begin{pmatrix} 1 & 1 & 3 \\ -1 & -2 & 0 \\ 2 & 3 & 4 \end{pmatrix}.$$

(c) Calculate the determinant of the matrix

$$\begin{pmatrix} 1 & 3 & -1 & 4 \\ 2 & 8 & 1 & 10 \\ -1 & -5 & -6 & 2 \\ 3 & 7 & -8 & 12 \end{pmatrix}.$$

3. (a) Let A be a $n \times n$ matrix. Explain what is meant by the statement that a column vector v is an *eigenvector* of A corresponding to the *eigenvalue* λ.
If v is an eigenvector of A corresponding to the eigenvalue λ, prove that

$$\det(\lambda I_n - A) = 0.$$

(Here I_n denotes the $n \times n$ identity matrix).

(b) Let $A = \begin{pmatrix} 5 & 7 \\ -1 & -3 \end{pmatrix}$.
Find an invertible 2×2 matrix E for which $E^{-1}AE$ is diagonal.

(c) Let $A = \begin{pmatrix} 8 & -7 & -7 \\ 2 & 1 & -1 \\ 7 & -9 & -7 \end{pmatrix}$. Find all the eigenvalues of A, and find an eigenvector of A corresponding to the eigenvalue $\lambda = -1$.

P.T.O

4. Students at the University of Wonderland like to relax on Friday evenings by going to the cinema, going to a concert or going to a football match. Every student participates in exactly one of these activities each Friday, according to the following pattern.

   - Of students who go to the cinema on a given week, 10% will go to the cinema the following week, 50% will go to a concert and 40% will go to a football match.
   - Of students who attend a concert on a given week, 50% will attend go to the cinema the following week, 20% will go to a concert, and 30% will go to a football match.
   - Of students who attend a football match on a given week, 30% will go to the cinema the following week, 30% will go to a concert, and 40% will go to a football match.

   (a) Write down the transition matrix for this Markov process, and explain why it has 1 as an eigenvalue.

   (b) Suppose that on Friday October 4, 40% of the students go to the cinema, 35% go to a concert, and 25% go to a football match. What percentage of the student population will go to a concert on Friday October 18?

   (c) Find the steady state of this Markov process.