

MA203 Linear Algebra 06/07 - Homework 4

Due date for problems marked (H) - Friday March 30

- (H) The population of a region is divided into rural and urban residents. Each year, 5% of the urban residents move to rural areas and 15% of rural residents move to urban areas.
 - Write down the transition matrix of the Markov process describing this situation.
 - If at the end of the year 2000, 60% of the population of the region were urban residents, what proportion of the population were urban residents at the end of 2002?
 - If this pattern of migration persists over many years, what proportion of the regions population will be rural residents in the long term?
- The subscription department of a magazine sends out letters annually to members of a large mailing list. Of the people receiving the letter, 60% of those who already subscribe to the magazine will renew their subscription, and 25% of those who do not already subscribe will begin to do so.
 - Write down the transition matrix of the Markov process describing this situation.
 - 40% of the recipients of last years letter ordered a subscription. What percentage of those who receive this year's letter can be expected to order a subscription?
- Find the steady state vector of a Markov process whose transition matrix is

$$\begin{pmatrix} 0.3 & 0.1 & 0.4 \\ 0.2 & 0.4 & 0.0 \\ 0.5 & 0.5 & 0.6 \end{pmatrix}.$$

- (H) Consider a plant that can have red flowers (R), pink flowers (P), or white flowers (W), depending on the genotypes RR, RW and WW. When we cross each of genotypes with the genotype RW, we obtain the transition matrix

		Flowers of parent plant		
		R	P	W
Flowers of offspring plant	R	$\begin{bmatrix} 0.5 & 0.25 & 0.0 \\ 0.5 & 0.5 & 0.5 \\ 0.0 & 0.25 & 0.5 \end{bmatrix}$		
	P			
	W			

Suppose that each successive generation is obtained by crossing only with plants of the RW genotype. When the process reaches equilibrium, what percentage of the plants will have red, pink, or white flowers?

5. A car rental company has three locations, A, B and C. The company's records over the years show that of cars rented from location A, 80% are returned to location A and 10% each are returned to locations B and C. Of cars rented from location B, 75% are returned to location B with 15% returned to location A and 10% returned to location C. Of cars rented from location C, 70% are returned to location C, and 15% each to locations A and B.
- (a) Write out the 3×3 matrix with Rows and Columns labelled by A,B,C, whose entry in a particular position is the proportion of cars rented from the location labelling the column that are returned to the location labelling the row.
- (b) If the pattern described above has now persisted for a long time, what proportion of the company's rentals are returned to location B?