MA203 Linear Algebra 06/07 - Homework 2

Due date for problems marked (H) - Friday February 23

1. (H) Decide if each of the following statements is true or false. Explain your answer in each case.

   (a) Every system of linear equations in which the coefficient matrix is square has a unique solution.

   (b) A system of equations with four equations and three variables can have a unique solution.

   (c) A system of linear equations with three equations and four variables cannot have a unique solution.

   (d) If A is a square matrix and there exists a system of equations having A as coefficient matrix and having a unique solution, then every system of linear equations having A as coefficient matrix has a unique solution.

   (e) A system of equations with three equations and four variables can be inconsistent.

2. Ann and Bob are participating in a mathematics contest which runs over five days (and has complicated rules). On each day each competitor must answer all the questions either on a white problem paper or a blue problem paper. Each white paper has five algebra problems and five calculus problems, and each blue paper has four algebra problems and six calculus problems. This information is summarized in the 2×2 matrix B defined as follows :

	Algebra	Calculus
White	5	5
Blue	4	6

$$B = \begin{pmatrix} 5 & 5 \\ 4 & 6 \end{pmatrix}.$$

   Over the five days, Ann completes three white papers and two blue papers, and Bob completes one white paper and four blue papers. This information is summarized in the 2×2 matrix A defined as follows :

	White	Blue
Ann	3	2
Bob	1	4

$$A = \begin{pmatrix} 3 & 2 \\ 1 & 4 \end{pmatrix}.$$

   (a) How many algebra questions are answered by Ann over the five days?

   (b) How many calculus questions are answered by Bob over the five days?

   (c) Compute the matrix product AB (where A and B are defined as above).

   (d) Explain the meaning of each entry of the product AB as a piece of information concerning Ann and Bob's participation in the contest. (For example you could do this by placing the entries of AB in a table similar to those used to define A and B themselves, and appropriately labelling the rows and columns.)

3. Let $A = \begin{pmatrix} 2 & -1 & 3 \\ -2 & 1 & -1 \end{pmatrix}$ and $B = \begin{pmatrix} 5 & 0 \\ 2 & -3 \\ -1 & 4 \end{pmatrix}$.
   Calculate the matrix products AB and BA.

4. (H) Let $A = \begin{pmatrix} 3 & 1 & -2 & 0 \\ 1 & 4 & -2 & 2 \\ -2 & 0 & 7 & -3 \\ -1 & 3 & -1 & 2 \end{pmatrix}$ and $B = \begin{pmatrix} 1 & 1 \\ 1 & -2 \\ -3 & 2 \\ 4 & 5 \end{pmatrix}$.
   Calculate the matrix product AB.

5. Let $A = \begin{pmatrix} 1 & 0 & 4 \\ 2 & -1 & 2 \end{pmatrix}$, $B = \begin{pmatrix} 1 & 0 & 1 \\ 2 & -1 & -2 \\ -3 & 2 & 0 \end{pmatrix}$, $C = \begin{pmatrix} 1 & -1 \\ 2 & 1 \\ -2 & 3 \end{pmatrix}$.

   (a) Calculate the products AB and BC.

   (b) Verify that $(AB)C = A(BC)$.

6. In $M_3(\mathbb{R})$, define

$$A = \begin{pmatrix} 3 & -2 & 1 \\ 2 & -1 & 2 \\ 1 & -2 & -4 \end{pmatrix} \qquad B = \begin{pmatrix} 8 & -10 & -3 \\ 10 & -13 & -4 \\ -3 & 4 & 1 \end{pmatrix}$$

   Show that A and B are inverses of each other.
   (Note : you can do this by just calculating the products AB and BA).

7. (H) Use Problem 6 above to find the unique solution to each of the following systems of equations.

$$\begin{array}{rcrcrcl} 3x & - & 2y & + & z & = & 2 \\ 2x & - & y & + & 2z & = & 3 \\ x & - & 2y & - & 4z & = & 1 \end{array} \qquad \begin{array}{rcrcrcr} 8x & - & 10y & - & 3z & = & 0 \\ 10x & - & 13y & - & 4z & = & -1 \\ -3x & + & 4y & + & z & = & 2 \end{array}$$

8. (H) Use elementary row operations to calculate the inverses of the following matrices, or to show that they are not invertible.

$$\text{(a)} \begin{pmatrix} 0 & 1 & 2 \\ 1 & 0 & 3 \\ 4 & -3 & 8 \end{pmatrix} \qquad \text{(b)} \begin{pmatrix} 1 & 0 & -2 \\ -3 & 1 & 4 \\ 2 & -3 & 4 \end{pmatrix}$$