MA203 Linear Algebra 06/07 - Homework 1
Due date for problems marked (H) - Wednesday February 7

1. Consider the following system of linear equations :

$$\begin{array}{rcrcrcr} x & - & y & + & 2z & = & 3 \\ 2x & - & y & + & z & = & 10 \\ x & + & y & - & 3z & = & 6 \end{array}$$

   (a) Write out the augmented matrix of this system.

   (b) Use elementary row operations to obtain a row-echelon form from this matrix.

   (c) Use the row-echelon form found in part (b) to find all solutions of the above system.

2. (H) Consider the following system of equations :

$$\begin{array}{rcrcrcr} 2x & - & y & - & 3z & = & -8 \\ x & + & 2y & + & z & = & 1 \\ 4x & - & 4y & - & 5z & = & -2 \end{array}$$

   (a) Write down the augmented matrix of this system.

   (b) Use elementary row operations to reduce this matrix to

      (i) row-echelon form

      (ii) *reduced* row-echelon form.

   (c) Using your answer(s) to (b) above, find all solutions of the system of equations.

3. (H) Consider the following system of equations :

$$\begin{array}{rcrcrcrcr} 3x_1 & + & x_2 & - & 2x_3 & + & x_4 & = & -8 \\ x_1 & - & 2x_2 & - & 2x_3 & + & 6x_4 & = & -3 \\ 2x_1 & - & x_2 & - & 3x_3 & + & 4x_4 & = & -7 \end{array}$$

   (a) Check that

      (i) $x_1 = -1$, $x_2 = -1$, $x_3 = 2$, $x_4 = 0$

      (ii) $x_1 = -11$, $x_2 = 14$, $x_3 = -3$, $x_4 = 5$

      (iii) $x_1 = 3$, $x_2 = -7$, $x_3 = 4$, $x_4 = -2$

      (iv) $x_1 = -101$, $x_2 = 149$, $x_3 = -48$, $x_4 = 50$

      are all solutions of this system.

   (b) Write down the augmented matrix of this system.

   (c) Use elementary row operations to reduce this matrix to

      (i) row-echelon form

      (ii) *reduced* row-echelon form.

(d) Using your answer to (ii) in (c) above, find the general solution of the system of equations.

(e) Find all solutions of the following system of equations :

$$\begin{aligned}
3x_1 + x_2 - 2x_3 + x_4 &= -8 \\
x_1 - 2x_2 - 2x_3 + 6x_4 &= -3 \\
2x_1 - x_2 - 3x_3 + 4x_4 &= -7 \\
x_1 + x_2 + x_3 + x_4 &= 3
\end{aligned}$$

4. (H) Consider the following system of equations :

$$\begin{aligned}
2x_1 - 4x_2 + x_3 - 2x_4 &= -2 \\
x_1 - 2x_2 - 2x_3 + 3x_4 &= -2 \\
3x_1 - 6x_2 - 4x_3 + 2x_4 &= -9
\end{aligned}$$

(a) Write down the augmented matrix of this system.

(b) Use elementary row operations to reduce this matrix to

   (i) row-echelon form
   (ii) *reduced* row-echelon form.

(c) Using your answer to (ii) in (b) above, find the general solution of the system of equations.

(d) Show that the system

$$\begin{aligned}
2x_1 - 4x_2 + x_3 - 2x_4 &= -2 \\
x_1 - 2x_2 - 2x_3 + 3x_4 &= -2 \\
3x_1 - 6x_2 - 4x_3 + 2x_4 &= -9 \\
x_1 - 2x_2 + x_3 + x_4 &= 0
\end{aligned}$$

is inconsistent.

5. For each of the following system of equations, find all solutions or show that none exist.

(a)
$$\begin{aligned}
x - 3y + 7z &= -11 \\
2x + y + 2z &= -1 \\
3x + 2y - 5z &= 0
\end{aligned}$$

(b)
$$\begin{aligned}
x - 3y + 7z &= -11 \\
2x + y &= -1 \\
2x - 3y + 8z &= -13
\end{aligned}$$

(c)
$$\begin{aligned}
x - 3y + 7z &= -11 \\
2x + y &= -1 \\
2x - 3y + 8z &= -13 \\
x + y + 2z &= 4
\end{aligned}$$

(d)
$$\begin{aligned}
x - 3y + 7z &= -11 \\
2x + y &= -1 \\
2x - 3y + 8z &= -1
\end{aligned}$$