MA180/190/186 Calculus Week 3

Supplementary Example on the substitution technique for integration (from Exercises 5.5 in Stewart's Calculus)

1. $$\int \frac{(\ln x)^2}{x}\,dx = \int (\ln x)^2 \underbrace{\boxed{\frac{1}{x}\,dx}}_{du}$$

Write $u = \ln x$

Then $du/dx = \frac{1}{x} \Rightarrow du = \boxed{\frac{1}{x}\,dx}$

Rewrite in terms of u

$$\int \underbrace{(\ln x)^2}_{u^2} \underbrace{\frac{1}{x}\,dx}_{du} = \int u^2\,du = \frac{u^3}{3} + C = \boxed{\frac{1}{3}(\ln x)^3 + C}$$

2. $$\int_e^{e^4} \frac{dx}{x\sqrt{\ln x}} = \int_e^{e^4} \frac{1}{\sqrt{\ln x}} \underbrace{\frac{1}{x}\,dx}_{du}$$

Try $u = \ln x$

Then $\frac{du}{dx} = \frac{1}{x} \Rightarrow du = \frac{1}{x}\,dx$

$$\int_e^{e^4} \underbrace{\frac{1}{\sqrt{\ln x}}}_{1/\sqrt{u}} \underbrace{\frac{1}{x}\,dx}_{du} = \int_{x=e}^{x=e^4} \frac{1}{\sqrt{u}}\,du$$

$u = \ln x$ $\quad x = e \Rightarrow u = \ln e = 1$

$x = e^4 \Rightarrow u = \ln e^4 = 4$

$$\int_{u=1}^{u=4} u^{-1/2}\,du = 2u^{1/2}\Big|_{u=1}^{u=4} = 2(2) - 2(1) = \boxed{2}$$

Note: the derivative of $1/x$ resembles $1/x^2$

3. $$\int_1^2 \frac{e^{1/x}}{x^2}\,dx$$

Try $u = 1/x = x^{-1}$ $\quad du/dx = -x^{-2} = -\frac{1}{x^2}$

$$\Rightarrow \quad du = -\frac{1}{x^2}\,dx \quad \Rightarrow \quad \frac{1}{x^2}\,dx = -du$$

$$\int_{x=1}^{x=2} e^u(-du) = -\int_{x=1}^{x=2} e^u\,du$$

Limits: When $x=1$, $u = 1/1 = 1$ $\quad$ When $x=2$, $u = 1/2$

$$-\int_{u=1}^{u=1/2} e^u\,du = -e^u\Big|_1^{1/2} = -e^{1/2} - (-e^1)$$
$$= e - \sqrt{e}$$

Remark Could also try $u = e^{1/x}$

Then $$du/dx = e^{1/x}\left(-1/x^2\right) = -\frac{e^{1/x}}{x^2}$$

$$du = -\frac{e^{1/x}}{x^2}\,dx \quad \Rightarrow \quad \frac{e^{1/x}}{x^2}\,dx = -du$$

Our problem is $$-\int_{x=1}^{x=2} du = -u\Big|_{x=1}^{x=2}$$

When $x=1$, $u = e^{1/1} = e$

$x = 2$, $u = e^{1/2} = \sqrt{e}$

$$-u\Big|_{u=e}^{u=\sqrt{e}} = -\sqrt{e} - (-e) = e - \sqrt{e}$$