Lecture 23: Exam Advice

- Six questions, two hours. Manage the time carefully.
- Same arrangements for MA180, MA190, MA186.
- The three calculus questions will match the three chapters of the lecture notes and the six homework sheets.
- This year each of the calculus questions will have four parts all worth equal marks.
- Read the questions carefully. Make sure you answer exactly what is asked, as clearly as you can.
- Think about the learning outcomes when you are preparing.
- Think about communicating to the person who will read your script that you have achieved the learning outcomes.

Sample Question 1

(a) Define a function A by

$$A(x) = \int_1^x \sqrt{t}\,dt$$

for $x \geq 1$.

i Draw a diagram that indicates the meaning of $A(x)$.

ii. What does the Fundamental Theorem of Calculus say about the function A?

iii. What is $A'(4)$?

Evaluate the following integrals.

(b) $\displaystyle\int_1^2 xe^{3x}\,dx$ (c) $\displaystyle\int \frac{\cos(\sqrt{x})}{\sqrt{x}}\,dx$ (d) $\displaystyle\int \frac{x^2+3x+3}{x+1}\,dx$

Sample Question 2

(a) State whether each of the following assertions is true or false. Give a short explanation (one or two lines) for your answer in each case.

  i. Every unbounded subset of $\mathbb{R}$ is infinite.
  ii Every infinite subset of $\mathbb{R}$ is unbounded.
  iii If a subset of $\mathbb{Q}$ is bounded above, then it has a maximum element.

(b) State what is means for an infinite set to be *countable*. Show that the set $\mathbb{Z}$ of integers is countable.

(c) Let $S = \left\{ \frac{2n+1}{n^2} : n \in \mathbb{Z},\ n \geq 1 \right\}$.

  1. List four elements of S.
  2. Show that S is bounded. Does S have a minimum element? Does S have a maximum element? Determine the infimum and supremum of S.

(d) Give an example of a bounded set of rational numbers whose infimum is irrational and whose supremum is rational.

Sample Question 3

(a) Give an example of

i. a convergent sequence of real numbers;

ii. a sequence of real numbers that is bounded and divergent;

iii. a sequence of real numbers that is convergent and is not monotonic.

(b) A sequence (a_n) of real numbers is defined by

$$a_0 = 4, \;\; a_n = \frac{1}{2}(a_{n-1} - 2) \text{ for } n \geq 1.$$

i. Write down the first four terms of the sequence.

ii. Show that the sequence is bounded below by -2.

iii. Show that the sequence is monotonically decreasing.

iv. State why it can be deduced that the sequence is convergent, and determine its limit.

(c) Show that the harmonic series $\displaystyle\sum_{n=1}^{\infty} \frac{1}{n}$ is divergent.

(d) Find the first three terms in the Maclaurin series of $e^{\cos x}$.