Example (similar to Q5 on sheet 61)

The sequence $(a_n)_{n=1}^{\infty}$ defined by

$$a_n = \max\left(\left\{0, \frac{(-1)^n}{2n}\right\}\right)$$

converges to 0. If $\varepsilon = \frac{1}{800}$, what is the least N for which $|a_n| < \varepsilon$ for all $n > N$?

- What are the terms of the sequence?

$n=1$: $a_1 = \max\left(\left\{0, -\frac{1}{2}\right\}\right) = 0$

$n=2$: $a_2 = \max\left(\left\{0, \frac{1}{4}\right\}\right) = \frac{1}{4}$

$n=3$: $a_3 = \max\left(\left\{0, -\frac{1}{6}\right\}\right) = 0$

$n=4$: $a_4 = \max\left(\left\{0, \frac{1}{8}\right\}\right) = \frac{1}{8}$

$0, \frac{1}{4}, 0, \frac{1}{8}, 0, \frac{1}{12}, 0, \frac{1}{16}, \ldots$

$a_n = 0$ when n is odd

$a_n = \frac{1}{2n}$ when n is even

- When do the terms get near to $\frac{1}{800}$ in absolute value?

$a_{398} = \frac{1}{796} > \frac{1}{800}$

* $a_{399} = 0$

* $a_{400} = \frac{1}{800} = \varepsilon$

$a_{401} = 0$

$a_{402} = \frac{1}{804} < \varepsilon$

all terms from this point on are $< \frac{1}{800}$

Answer: $N = 400$