

- Six questions, two hours. Manage the time carefully.
- Same arrangements for MA180, MA190, MA186.
- The three calculus questions will match the three chapters of the lecture notes and the six homework sheets.
- This year each of the calculus questions will have four parts all worth equal marks.
- Read the questions carefully. Make sure you answer exactly what is asked, as clearly as you can.
- Think about the learning outcomes when you are preparing.
- Think about communicating to the person who will read your script that you have achieved the learning outcomes.

Sample Question 1

- (a) Define a function A by

$$A(x) = \int_1^x \sqrt{t} dt$$

for $x \geq 1$.

- i. Draw a diagram that indicates the meaning of $A(x)$.
 - ii. What does the Fundamental Theorem of Calculus say about the function A ?
 - iii. What is $A'(4)$?
- (b) Evaluate the following integrals.

i. $\int_1^2 xe^{3x} dx$ ii. $\int \frac{\cos(\sqrt{x})}{\sqrt{x}} dx$ iii. $\int \frac{x^2 + 3x + 3}{x + 1} dx$

Sample Question 2

- (a) State whether each of the following assertions is true or false. Give a short explanation (one or two lines) for your answer in each case.
- Every unbounded subset of \mathbb{R} is infinite.
 - Every infinite subset of \mathbb{R} is unbounded.
 - If a subset of \mathbb{Q} is bounded above, then it has a maximum element.
- (b) State what is means for an infinite set to be *countable*. Show that the set \mathbb{Z} of integers is countable.
- (c) Let $S = \left\{ \frac{2n+1}{n^2} : n \in \mathbb{Z}, n \geq 1 \right\}$.
- List four elements of S .
 - Show that S is bounded.
 - Does S have a minimum element? Does S have a maximum element? Determine the infimum and supremum of S .

Sample Question 3

- (a) Give an example of
- a convergent sequence of real numbers;
 - a sequence of real numbers that is bounded and divergent;
 - a sequence of real numbers that is convergent and is not monotonic.
- (b) A sequence (a_n) of real numbers is defined by

$$a_0 = 4, \quad a_n = \sqrt{a_{n-1}^2 - 2a_{n-1} + 4} \text{ for } n \geq 1.$$

- Write down the first four terms of the sequence.
 - Show that the sequence is bounded below by 2.
 - Show that the sequence is monotonically decreasing.
 - State why it can be deduced that the sequence is convergent, and determine its limit.
- (c) Find the first four terms in the Maclaurin series of $\frac{1}{1-x}$.