MA313 24 Sep
Assignment 1, Q6

All linear combinations of u and v are $au + bv$ for some $u, v \in \mathbb{R}$.
Note $u = 0$ or $v = 0$ is OK.

$u = \begin{bmatrix} -6 \\ -3 \end{bmatrix}$ $v = \begin{bmatrix} 24 \\ 8 \end{bmatrix}$

① Is $3\begin{bmatrix} -6 \\ -3 \end{bmatrix}$ a **linear** combination of u & v?
Yes, since $3\begin{bmatrix} -6 \\ -3 \end{bmatrix} = 3u + 0v$

② Is $1\begin{bmatrix} 24 \\ 8 \end{bmatrix} + 3\begin{bmatrix} -6 \\ -3 \end{bmatrix}$ a linear combination of u&v? Yes.

③ Is $\begin{bmatrix} -6 \\ -3 \end{bmatrix}$ a linear combination? Yes! $((1)u + (0)v)$

④ Is $\begin{bmatrix} 0 \\ 0 \end{bmatrix}$ a linear combination of u & v? Yes! $(0)u + (0)v = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$

⑤ Are all vectors in $\mathbb{R}^2$ a linear combination of u and v?
Any vector $\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} \in \mathbb{R}^2$ would have to be written as
$\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = a\begin{bmatrix} -6 \\ -3 \end{bmatrix} + b\begin{bmatrix} 24 \\ 8 \end{bmatrix}$ for some a, b. If we can solve for a and b, the answer is Yes.

That is
$$-6a + 24b = x_1 \Rightarrow -\cancel{6a} + 24b = x_1$$
$$-3a + 8b = x_2 \Rightarrow \cancel{6a} - 16b = -2x_2$$
$$8b = x_1 - 2x_2$$

So $b = \dfrac{x_1 - 2x_2}{8}$. Then use $a = \dfrac{x_2 - 8b}{-3} = \dfrac{x_2 - x_1 + 16x_2}{8} = \dfrac{-x_1 + 17x_2}{8}$

Show that not all vectors in $\mathbb{R}^2$ are linear combinations of $\begin{bmatrix} -6 \\ -3 \end{bmatrix}$ and $\begin{bmatrix} 24 \\ 12 \end{bmatrix}$.

Suppose $\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = a\begin{bmatrix} -6 \\ -3 \end{bmatrix} + b\begin{bmatrix} 24 \\ 12 \end{bmatrix}$

So
$$-6a + 24b = x_1 \Rightarrow -6a + 24b = x_1$$
$$-3a + 12b = x_2 \quad\quad -6a - 24b = -2x_2$$
$$0 + 0 = x_1 - 2x_2$$

Can't solve for a and b!

Q3 is $w = \begin{bmatrix} 3 \\ 3 \\ 1 \end{bmatrix}$ a linear combination of

$$u_1 = \begin{bmatrix} -2 \\ -3 \\ 4 \end{bmatrix} \quad u_2 = \begin{bmatrix} -6 \\ -10 \\ 12 \end{bmatrix} \quad u_3 = \begin{bmatrix} -19 \\ -29 \\ 36 \end{bmatrix}$$

That is, can we solve $a u_1 + b u_2 + c u_3 = w$

$$a \begin{bmatrix} -2 \\ -3 \\ 4 \end{bmatrix} + b \begin{bmatrix} -6 \\ -10 \\ 12 \end{bmatrix} + c \begin{bmatrix} -19 \\ -29 \\ 36 \end{bmatrix} = \begin{bmatrix} 3 \\ 3 \\ 1 \end{bmatrix}$$

The Eqn's are

$$\begin{aligned} -2a - 6b - 19c &= 3 \\ -3a - 10b - 29c &= 3 \\ 4a + 12b + 36c &= 1 \end{aligned}$$

Now apply Gaussian Elim to solve for a, b and c.