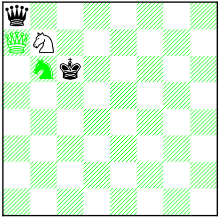
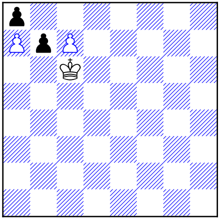




MA204/MA284 : Discrete Mathematics

Week 3: Binomials Coefficients

Dr Niall Madden

22 & 24 September 2021

1. Part 1: Bit strings and latice paths
   - An "Investigate" activity
   - Bit strings
   - Lattice Paths
2. Part 2: Binomial coefficients
   - Calculating binomial coefs
3. Part 3: Pascal's triangle
4. Part 4: Permutations
   - Examples
   - The binomial coefficient formula
5. Exercises

These slides are based on §1.2 of Oscar Levin's *Discrete Mathematics: an open introduction*. They are licensed under CC BY-SA 4.0

Tutorials started this week! You should attend one of the sessions listed below.

	Mon	Tue	Wed	Thu	Fri
9 – 10					
10 – 11					
11 – 12			CD: **MRA201**		
12 – 1		EM: CA117			
1 – 2					
2 – 3			AH Online		
3 – 4		AH: Online		CD: Online	
4 – 5				EM: **AMB-G008**	

Online class will be held on the course room in the Blackboard Virtual Classroom: eu.bbcollab.com/guest/768da44b88344e86bf5eae54357e2be9

Also, today (Wednesday, 22 Sep) is your last chance to indicate an alternative time for an in-person tutorial:
`https://forms.office.com/r/9uBcpERuqy`

ASSIGNMENT 1 is now open!

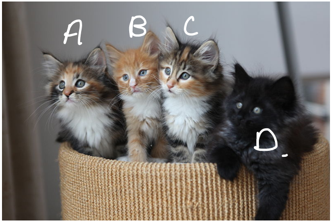
To access the assignment, go to the 2122-MA284 Blackboard page, select **`Assignments`** ... **`Assignment 1`**.

There are 10 questions.
You may attempt each one up to 10 times.
This assignment contributes approximately 8% to your final grade for Discrete Mathematics.

Deadline: 5pm, Friday 1 October 2021.

$(1, 4, 6, 4, 1)$

$n=4$



Source: http://www.flickr.com/photos/35652310@N00/4139577452/.

$\{AB, AC, AD, BC, BD, CD\}$.

Example

The NUIG Animal Shelter has 4 cats.

(a) How many choices do we have for a single cat to adopt? $n=4,\ K=1$

$\binom{4}{1} = 4$

(b) How many choices do we have if we want to adopt two cats? $\binom{4}{2} = \binom{3}{1} + \binom{3}{2}$

(c) How many choices do we have if we want to adopt three cats? $\binom{4}{3} = 4$

(d) How many choices do we have if we want to adopt four cats? 1

$$\binom{4}{2} = \binom{3}{1} + \binom{3}{2} = 3 + 3 = 6.$$

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END OF PART 3

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Week 3: Binomials Coefficients

Start of ...

PART 4: Permutations

Order matters

(Combination – order does not Matter).

A **permutation** is an arrangement of objects. Changing the order of the objects gives a different permutation.

Example: List all permutations of the letters A, R and T?

ART RAT TAR

ATR RTA TRA.

Important: order matters - "ART" $\neq$ "TAR" $\neq$ "RAT".

ordering.

A **permutation** is an arrangement of objects. Changing the order of the objects gives a different permutation.

We can also count the number of permutations of the letters A, R and T, without listing them:

Use the multiplicative Prin:

3 choices for the first letter.

2 choices for " 2nd "

1 choice " " 3rd letter.

By the Multiplicative Prin: Answer is $3 \times 2 \times 1 = 6$

More generally, recall that $n!$ (read "n factorial") is

$$n! = n \times (n-1) \times (n-2) \times \cdots \times 2 \times 1$$

E.g.,

$$0! = 1, \quad 1! = 1, \quad 2! = 2, \quad 3! = 6, \quad 4! = 24, \quad 5! = 120, \quad 6! = 720.$$

$$10! = 3{,}628{,}800, \quad 20! = 2{,}432{,}902{,}008{,}176{,}640{,}000 \approx 2.43 \times 10^{18}.$$

Number of permutations

There are

$$n! = n \times (n-1) \times (n-2) \times \cdots \times 2 \times 1$$

(i.e., n factorial) permutations of n (distinct) objects.

no 2 the same.

To emphasis the **order matters** in permutations, consider the following example.

Example

In the recent paralympics, **8** athletes contested the Men's Va'a 200m Singles's final. How many different finishing orderings were possible?

(Sam Barnes/Sportsfile)

Ans: $8!$

$= 8 \times 7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1$.

Permutations of k objects from n

The number of permutations of k objects out of n, $P(n, k)$, is

$$P(n,k) = n \times (n-1) \times \cdots \times (n-k+1) = \frac{n!}{(n-k)!}$$

$$\frac{n!}{(n-k)!} = \frac{n(n-1)(n-2)\cdots(n-k+1)\,\cancel{(n-k)}\,(n-\cancel{k-1})\cdots\cancel{(2)}\cancel{(1)}}{\cancel{(n-k)}\,(n-\cancel{k-1})\cdots\cancel{(2)}\cancel{(1)}}$$

$$= n(n-1)(n-2)\cdots(n-k+1).$$

Example ($P(7,3)$)

In the recent paralympics, **8** athletes contested the Men's Va'a 200m Singles's final. In how many different ways could the gold, silver, and bronze medals be awarded? $n = 8$ $k = 3$

Sea Forest Waterway - 4 Sep - 11:56 - Official

Race Results

Rank	Name	Time	Time Behind
1	AUS McGRATH Curtis Sport Class: VL3	50.537	
2	BRA VIEIRA de PAULA Giovane Sport Class: VL3	52.148	+1.611
3	GBR WOOD Stuart Sport Class: VL3	52.760	+2.223
4	UZB SHERKUZIEV Khaytmurot Sport Class: VL3	52.793	+2.256
5	IRL O'LEARY Patrick Sport Class: VL3	52.910	+2.373
6	FRA POTDEVIN Eddie Sport Class: VL3	53.055	+2.518
7	BRA RIBEIRO de CARVALHO Caio Sport Class: VL3	53.246	+2.709
8	NZL MARTLEW Scott Sport Class: VL3	54.756	+4.219

Ans: $\frac{8!}{5!}$

$= 8 \times 7 \times 6.$

Choosing the "back three" on a rugby team...

Ireland Squad for the Women's Rugby World Cup Qualifiers has 5 players who (we'll say) call all play on the Left Wing (11), Right Wing (14) or Full-Back (15):

Amee-Leigh Murphy-Crowe • Eimear Considine • Lauren Delany
Beibhinn Parsons • Lucy Mulhall

1. How many choices do we have for picking the starting back three, without assigning them numbers? $\binom{5}{3} = 10.$ Combination

   

   Beibhinn Parsons scoring against Italy last Sunday

2. How many choices for picking a starting 11, 14 and 15 (i.e., numbers are assigned)? $5 \times 4 \times 3 = 60.$ Permutation.

Still choosing the back three...

Our rugby squad has 5 backs that can play at 11, 14, or 15.

There are $\binom{5}{3}$ ways we can pick 3 of them for our starting team, without allocating numbers.

Once we have picked these three, there are $3! = 6$ ways we can assign them the 11, 14 and 15 jerseys. That is

$$P(5,3) = \binom{5}{3} 3!.$$

However, we know $P(5,3)$, so this gives a formula for $\binom{5}{3}$.

(1) We know there are $P(n, k)$ permutations of k objects out of n.

(2) We know that

$$P(n, k) = \frac{n!}{(n-k)!}$$

(3) Another way of making a permutation of k objects out of n is to

(a) Choose k from n without order. There are $\binom{n}{k}$ ways of doing this.

(b) Then count all the ways of ordering these k objects. There are $k!$ ways of doing this.

(c) By the Multiplicative Principle,

$$P(n, k) = \binom{n}{k} k!$$

(4) So now we know that $\dfrac{n!}{(n-k)!} = \dbinom{n}{k} k!$

(5) This gives the formula $\dbinom{n}{k} = \dfrac{n!}{(n-k)!k!}$

Q1. Let $S = \{1, 2, 3, 4, 5, 6\}$

(a) How many subsets are there total?
(b) How many subsets have $\{2, 3, 5\}$ as a subset?
(c) How many subsets contain at least one odd number?
(d) How many subsets contain exactly one even number?
(e) How many subsets are there of cardinality 4?
(f) How many subsets of cardinality 4 have $\{2, 3, 5\}$ as a subset?
(g) How many subsets of cardinality 4 contain at least one odd number?
(h) How many subsets of cardinality 4 contain exactly one even number?

Q2. How many subsets of $\{0, 1, \ldots, 9\}$ have cardinality 6 or more? (Hint: Break the question into five cases).

Q3. How many shortest lattice paths start at (3,3) and end at (10,10)?
How many shortest lattice paths start at (3,3), end at (10,10), and pass through (5,7)?

Q4. Suppose you are ordering a large pizza from *D.P. Dough*. You want 3 distinct toppings, chosen from their list of 11 vegetarian toppings.

(a) How many choices do you have for your pizza?
(b) How many choices do you have for your pizza if you refuse to have pineapple as one of your toppings?
(c) How many choices do you have for your pizza if you *insist* on having pineapple as one of your toppings?
(d) How do the three questions above relate to each other?