MA284 : Discrete Mathematics

Week 1: Intro to Discrete Mathematics; The Additive and Multiplicative Principles

Niall Madden (Niall.Madden@NUIGalway.ie)

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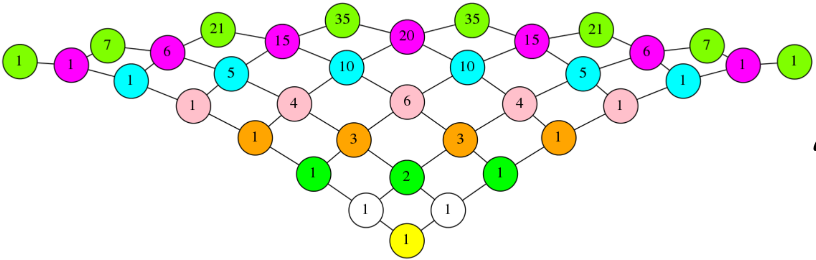


Reminder: please keep your video and audio turned off during the class. Please DO use the chat facility: that will not be recorded.

MA284
Week 1: Intro to Discrete Mathematics; The Additive and Multiplicative Principles

Start of ...

PART 3: Counting

Combinatorics is the mathematics of *counting*. It is an ancient field of study, though its "modern" history began with the systematic study of gambling in the 17^{th} century.

The simplest method of counting is *simple enumeration* = *"Point and count"*.

1. How many students in this calls have a last name that begins with `A`?

   Ans : 4.

2. How many anagrams are there of the letters **`NUI`**?

   INU NIU UIN Ans : 6.
   IUN NUI UNI

   Usually we don't want to make a list of all possibilities:

3. How many car licence plates are there of the form `XXX-yyy`, where X is a letter and y is a digit?

   Answer: There are 17,576,000, but we don't want to list them all.

The first techniques that we will study for solving counting problems are called

The Additive and Multiplicative Principles

For more information see Chapter 1 (Counting) of Oscar Levin's *Discrete Mathematics: an open introduction*.

Free.

1. There are 5 starters and 6 main-courses on a restaurant's menu. How many choices do you have if
   - (a) You would like one dish: a starter *or* a main-course?
   - (b) You would like two dishes: a starter **and** a main-course?

Answer : $5 + 6 = 11$. (additive Prin)

Answer : $5 \times 6 = 30$ (Mult Prin)

2. A standard deck of cards has 26 red cards, and 12 face/court cards.
   - (a) How many ways can you select a card that is red *and* face card?
   - (b) How many ways can you select a card that is red *or* face card?

Think about these questions as we go through the next sections.

MA284
Week 1: Intro to Discrete Mathematics; The Additive and Multiplicative Principles

END OF PART 3

MA284
Week 1: Intro to Discrete Mathematics; The Additive and Multiplicative Principles

Start of ...

PART 4: The Additive Principle

Example

The NUIG Animal Shelter has 4 cats and 6 dogs in need of a home. You would like a new pet (but just one!). How many choices do you have?

Ans: $4+6=10.$

The Additive Principle

If event A can occur m ways, and event B can occur n (disjoint) ways, then event "A **or** B" can occur in $m+n$ ways.

Example

1. Can we use the additive principle to determine how many two letter "words" **begin with** either A **or** B?

Yes: These "words" are AA, AB, AC, ...
BA, BB, BC, BD, ...

> They are distinct

2. Can we use the additive principle to determine how many two letter "words" **contain** either A **or** B?

No: The words containing A are
AA, AB, BA, AC, CA, AD, DA, ...
with B: BA, AB, BB, BC, CB, BD, DB, ...

Not distinct.

"Event": "a thing that can happen".

The Additive Principle

If event A can occur m ways, and event B can occur n (disjoint) ways, then event "A **or** B" can occur in $m+n$ ways.

Example

The NUIG Animal Shelter has 4 cats, 6 dogs, and 7 donkeys in need of a home. How many choices do you have for a new pet?

This generalises to multiple events.

Ans: $4+6+7 = 17$.

The Additive Principle

If event A can occur m ways, and event B can occur n **disjoint** ways, then event "A **or** B" can occur in $m + n$ ways.

Example

A deck of cards has 26 red cards and 12 "face"-cards.

1. How many ways can you pick a red card? — 26.
2. How many ways can you pick a face-card? Ans: 12.
3. How many ways can you pick a card that is red *or* is a face-card?

Answer is not $26+12 = 38$ (it is actually 32).

This last example is important because it emphasises the importance of the sets being **disjoint**.

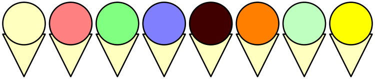
MA284
Week 1: Intro to Discrete Mathematics; The Additive and Multiplicative Principles

END OF PART 4

MA284
Week 1: Intro to Discrete Mathematics; The Additive and Multiplicative Principles

Start of ...

PART 5: The Multiplicative Principle



Example

Your favourite ice-cream shop has **8** flavours of ice-cream.
You can also choose between a cone, a waffle, and a cup.
How many choices to you have?

Ans : $8 * 3 = 24$..

The Multiplicative Principle

If event A can occur m ways, and each possibility allows for B to occur in n (disjoint) ways, then event "A and B" can occur in $m \times n$ ways.

Example

The NUIG Animal Shelter has 4 cats and 6 dogs in need of a home. How many choices do you have if you would like a cat and a dog as pets?

Answer: $4 \times 6 = 24$.

Example

The NUIG Animal Shelter also has 7 donkeys. How many choices to you have if you want a cat, a dog and a donkey?

Answer: $4 \times 6 \times 7 = 168$.

MA284
Week 1: Intro to Discrete Mathematics; The Additive and Multiplicative Principles

END OF PART 5

MA284
Week 1: Intro to Discrete Mathematics; The Additive and Multiplicative Principles

Start of ...

PART 6: Counting with sets

A set is a collection of things. The items in a set are called *elements*.
Examples:

- The set of natural numbers from 1 to 10 is $\{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$
- The set of upper-case letters is $\{A, B, \ldots, Y, Z\}$
- The set of students registered for Discrete Mathematics has 224 elements.
- A set is *unordered*.

ie $\{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$ same

as $\{10, 2, 1, 3, 5, 8, 9, 4, 6, 7\}$.

You should be familiar with the following basic elements of set notation:

subset . union intersection size

$\{\cdot\}$ $\in$ $\notin$ $\subseteq$ $\cup$ $\cap$ $\emptyset$ $|\cdot|$ $\setminus$ — "set diff".

"is an element of. empty

Example

Let $A = \{1,2,3\}$, $B = \{1,3,5\}$, and $C = \{2,4\}$.

- $2 \in A$, ✓ $4 \notin A$ ✓
- $\{1,3\} \subseteq A$ ✓
- $A \cup B = \{1,2,3,5\}$
- $A \cap B = \{1,3\}$, $B \cap C = \emptyset$, ✓
- $|A| = 3$, $|B \cap C| = 0$,
- $A \setminus B = \{2\}$ $A \setminus C = \{1,3\}$

Also, for any set X, $X \subseteq X$ $\emptyset \subseteq X$.

Let's return to the restaurant problem again, changed slightly...

We have a choice of 3 deserts = $\{a, b, c\}$.

a = "apple pie"
b = "banana split"
c = "choc cake"

Choice of 4 drinks:

$\{T, \varepsilon, L, W\}$.

T = tea
ε = espresso
L = Latte
W = wine.

Let's return to the restaurant problem again, changed slightly...

Deserts : $\{a, b, c\}$
Drinks : $\{t, e, l, w\}$.

To choose one item : we have
7 choices : $|\{a, b, c\}| + |\{t, e, l, w\}|$
$3 + 4$.

To choose one desert & one drink ?

(a, t), (a, e), (a, l), (a, w)
(b, t), (b, e), (b, l), (a, w)
(c, t), (c, e), (c, l), (c, w).

Ans : 12.

MA284
Week 1: Intro to Discrete Mathematics; The Additive and Multiplicative Principles

END OF PART 6

MA284
Week 1: Intro to Discrete Mathematics; The Additive and Multiplicative Principles

Start of ...

PART 7: Exercises

Here are a set of exercises to help you work through the material presented during this week's classes.

All but the last are taken either directly from the textbook, or with minor edits.

You do not have to submit your solutions to be graded.

0. Read Chapter 0 of Levin's *Discrete Mathematics: an open introduction* from `http://discretetext.oscarlevin.com`.
   Do Exercises 1–9 in Chapter 0 (these are interactive, with hints and solutions).
1. Your wardrobe consists of 5 shirts, 3 pairs of pants, 17 bow ties, and one fez (hat). How many different outfits can you make?
2. For your job interview at the NUIG Animal Shelter, you must wear a tie. You own 3 regular (boring) ties and 5 (cool) bow ties. How many choices do you have for your neck-wear?

3. You realise that the interview is actually for ClownSoc, so you should probably wear both a regular tie and a bow tie. How many choices do you have now?

4. Your DVD collection consists of 9 comedies and 7 horror movies. Give an example of a question for which the answer is:
   (a) 16.
   (b) 63.

5. If $|A| = 10$ and $|B| = 15$, what is the largest possible value for $|A \cap B|$? What is the smallest? What are the possible values for $|A \cup B|$?

6. If $|A| = 8$ and $|B| = 5$, what is $|A \cup B| + |A \cap B|$?