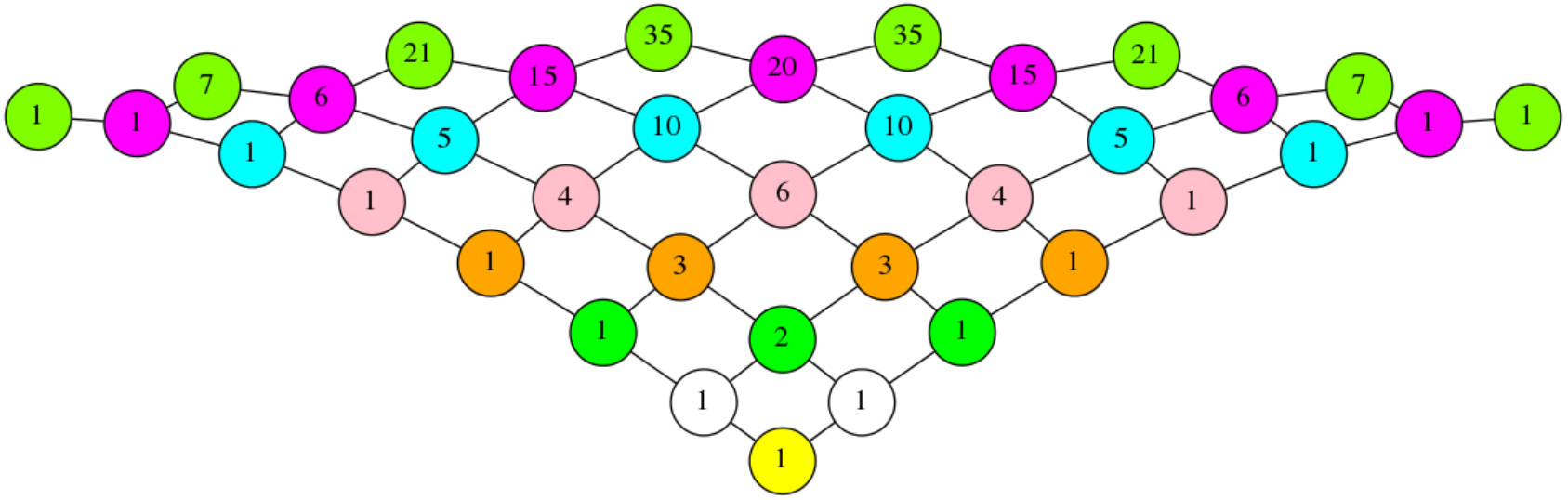
MA284 : Discrete Mathematics

Week 1: Intro to Discrete Mathematics; The Additive and Multiplicative Principles

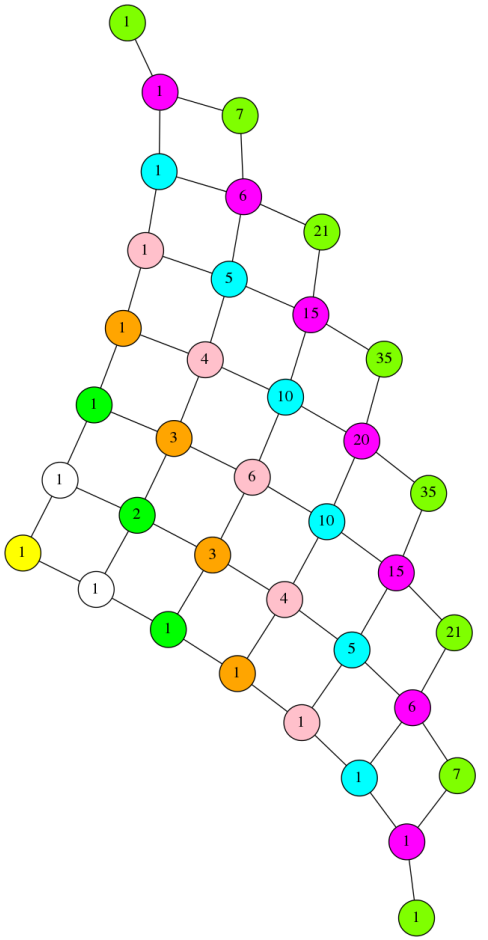
Niall Madden (Niall.Madden@NUIGalway.ie)

8 & 10 September, 2021



Reminder: please keep your video and audio turned off during the class. Please DO use the chat facility: that will not be recorded.

Outline



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Week 1: Intro to Discrete Mathematics; The Additive and Multiplicative Principles

Start of ...

PART 1: All about MA284

Lecturer: Dr Niall Madden (he/him),
School of Mathematics, Statistics and Applied Mathematics, NUIG.
Room ADB-1013, Arás de Brún
Email: Niall.Madden@NUIGalway.ie,

The best way to contact me is by email. + or discussion board or Blackboard

https://commons.wikimedia.org/wiki/File:%C3%81ras_de_Br%C3%BAn.jpg

This module is taken by about 224 (and counting) students in

- 2nd Science: Mathematics, Mathematical Science, Physics, E&O, Computer Science, Financial Mathematics and Economics, ...;
- Arts: 2nd Mathematics, 2nd Music(!), 3rd Mathematics/Computer Science & Education, Data Science...;
- 2nd Computer Science & IT (2BCT1);
- Visiting student(s).

Given your *very* varied backgrounds, you will need to stay focused, and become practised at communicating your own insights and challenges...

This is *Discrete Mathematics*: a mathematics module introduces the concepts of

- *enumerative combinatorics*: how to count, – weeks 1 – 6
- *graph theory* (i.e., the theory of graphs). – weeks 7 – 12

Don't worry: most of the rest of the definitions in this module will be more helpful than that!

Lectures: Wednesdays, **13.00-13.50+** ONLINE at `https://eu.bbcollab.com/guest/aa5b2975489749 8d811dea8c991d7d3b`, with videos posted later that evening.
Fridays, **11–11.45** in person on the O'Flaherty Lecture Theatre. If the technology allows, it too will be live-streamed, and videos will be made available.

Tutorials: They will start in Week 3. More details in a moment.

Blackboard: You will find lots of resources on Blackboard

- Announcements;
- These slides;
- Videos
- Grades;
- Link to the textbook,
- Access to assignments.

Work load: 5 ECTS (60 is the typical total for a full-time programme)
24 lectures, all in Semester 1
Roughly 120 hours of student effort time.

Lecture materials: Slides for the week's classes will be available for download in advance of the Wednesday lecture. Please let me know if you spot typos.
The slides contain the main definitions, ideas, and examples. Examples that are worked out in class will be posted later in the week.
Each set of slides finishes with a list of exercises, which are of a similar style and standard as those on the final exam.

Images: Particularly in the second half of this module, there will be lots of pictures of graphs. These are mostly generated using **Graphviz** `http://www.graphviz.org/` and/or NetworkX `https://networkx.github.io/`
I'll make the source code available. But if I forget, please ask!

SUMS: The School of Maths provides a free drop-in centre called

SUMS: Support for Undergraduate Maths Students.

SUMS opens from **2pm to 5pm, Monday to Friday**, from Monday of Week 3. For more information, see
http://www.maths.nuigalway.ie/sums/

Devices: The use of portable electronic devices during class is *encouraged*. For example, you might want to use it to check Wikipedia, or access the textbook.
Be aware that these can be distracting to other students. Please be considerate.

Other stuff: Today is **Soc's Day**! Why not (re)join the Mathematics Society? https://www.facebook.com/MathsSocNUIG
Also, consider joining our Student Chapter of SIAM:
http://www.maths.nuigalway.ie/SIAM-Galway/

MA284 will be assessed as follows.

Continuous assessment: There will be **five** online assignments, together worth 40% of the final grade.
Multiple attempts can be made, and scoring (right/wrong) is provided immediately. These will help you test yourself, and give you time to seek support at tutorials.

WeBWorK: The Online Assignments uses "**WeBWorK**", the same system as the interactive exercises in the text-book (more about that in a minute). You access the assignments through Blackboard.

Final assessment: There will be a 2 hour exam at the end of the semester, worth **60%**.

Tutorials will start in Week 3 (week beginning **20** September).You should attend *one tutorial per week*.

The tentative arrangements for this year below. All arrangements are tentative for now.

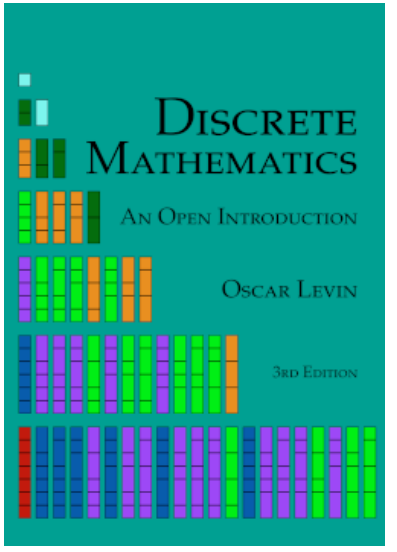
	Mon	Tue	Wed	Thu	Fri
9 – 10					
10 – 11					
11 – 12			? AdB-G021 ?		
12 – 1		? CA117 ?			
1 – 2					
2 – 3			? AC215 ?		
3 – 4		? AC213 ?		? AdB1020 ?	
4 – 5			? IT203 ?		

A survey will be circulate later to see if these times suit, and if there is a preference for them to be in person or on-line.

Email Niall if none of these times work for you.

The main recommended text is

Oscar Levin, Discrete Mathematics: an open introduction, 3rd Edition. This is a free, open source textbook, available from `http://discretetext.oscarlevin.com`*, in both printable and tablet/ereader-friendly versions. It is published under Creative Commons (CC BY-SA 4.0)*



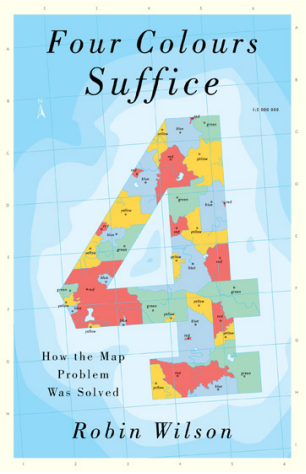
Other recommended texts include:

- Normal L Biggs, *Discrete Mathematics*, Oxford Science Publications. There are about 10 copies in the library at `510 BIG`.
- Kenneth Rosen, *Discrete Mathematics and Its Applications*, McGraw-Hill. Located at `511 ROS`.

Other books and resources will be mentioned through the semester.

Some related, fun, reading.

Really Big Numbers, by Richard Schwartz, published by the American Mathematical Society.
It is aimed a children, but is quite sophisticated. So you can learn some Discrete Mathematics while doing bed-time reading! It is in the library at 513 SCH
Watch at
https://www.youtube.com/watch?v=cEOY9UAsCFM



Four Colors Suffice: How the Map Problem Was Solved.
Robin Wilson.
In the library at 511.5 WIL
This is the story of the solution of one of most famous mathematical problems, that defied solution for nearly 150 years. It is also a treatise on what "proof" really means.

Do you have any other suggestions?

There are very few prerequisites for this module. I will expect that

- you can reason logically;
- understand the concept of a *proof*, as know several proof techniques, such as *induction*.
- know what a matrix is, and how to multiply a matrix by a vector, and a matrix by a matrix.
- you are comfortable with the concept of **sets**, and the notation used to describe and manipulate them.
- you are comfortable with the concept of **functions**, and the notation used to describe and manipulate them.

Exercise

Read Sections 0.3 (Sets) and 0.4 (Functions) in Chapter 0 of *Discrete Mathematics: an open introduction*

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END OF PART 1

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PART 2: What is Discrete Mathematics

If calculus is "continuous mathematics", then "discrete mathematics" is everything else! However, it is usually taken to include the following

1. Logic
2. Sets and set-theory;
3. Mathematics of Algorithms;
4. Recursion and induction;
5. Counting;
6. Discrete probability;
7. Graphs, trees and networks;
8. Boolean algebra;
9. Modelling computing (Turing machines and Finite State Machines).

But we will just focus on **counting (combinatorics)** and *graphs*.

1. **Combinatorics**.

How to count,

The additive and multiplicative principles.

The Binomial coefficients and some identities.

The principal of Inclusion-Exclusion.

Permutations and Combinations.

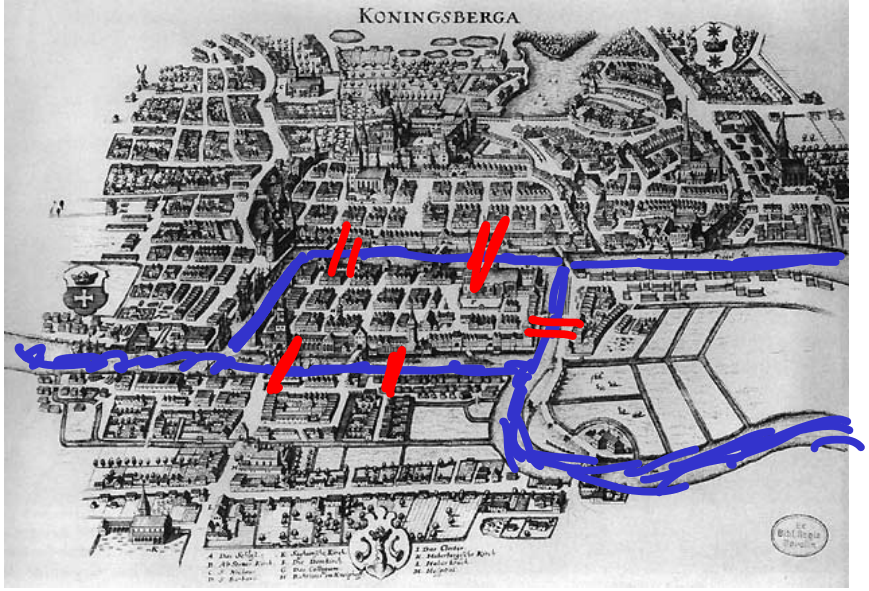
Non-negative equations and inequalities.

Derangements and distributions

including Pascal's Triangle.

2. **Graph Theory**.

Euler and the Koenigsberg Bridges Problem.
Eulerian and Hamiltonian graphs.
Tree graphs and bipartite graphs.
Planarity of Graphs.
Euler's formula for a connected planar graph.
Planarity and the Platonic solids;
Colouring of Graphs.



Combinatorics has an ancient history. The earliest known is in a 3,500 year old Egyptian manuscript. It posed a question like *"In 7 houses are 7 cats, each with 7 mice, who each have 7 heads of wheat, which each have 7 grains. How many houses, cats, mice, heads of wheat and grains are there?*

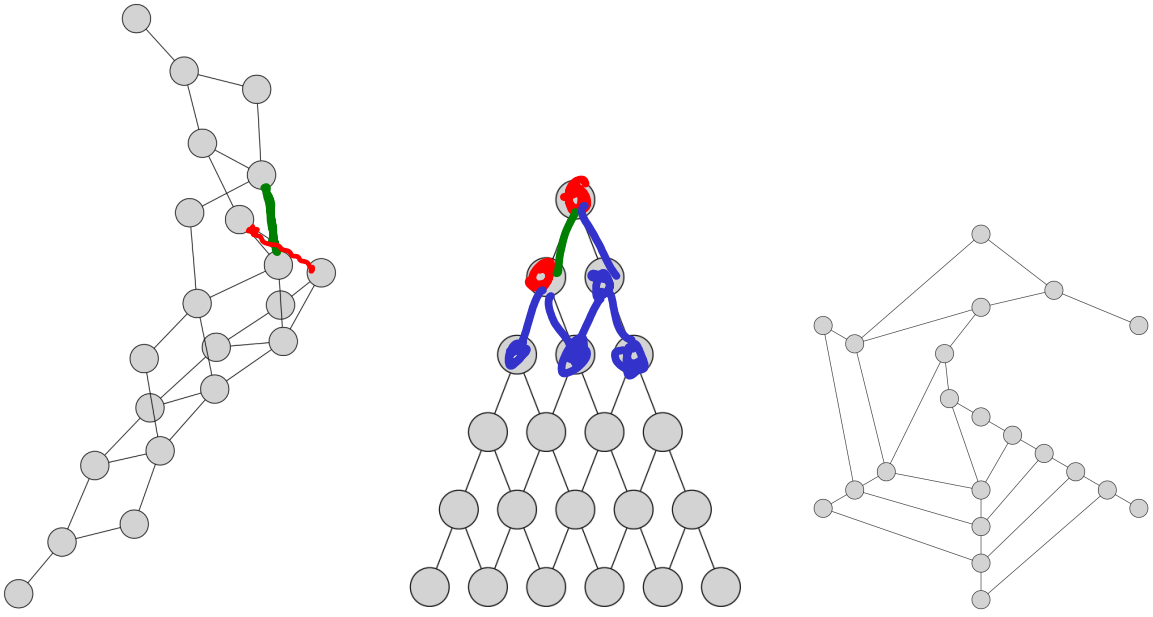
Description: The so-called "Rhind Mathematical Papyrus" : detail (British Museum, EA10057)

Source: http://www.archaeowiki.org/Image:Rhind_Mathematical_Papyrus.jpg

Slightly more recently, in the 6th century the Indian physician Sushruta determined that there are $2^6 - 1 = 63$ different combinations of the tastes *sweet*, *pungent*, *astringent*, *sour*, *salt*, and *bitter*.

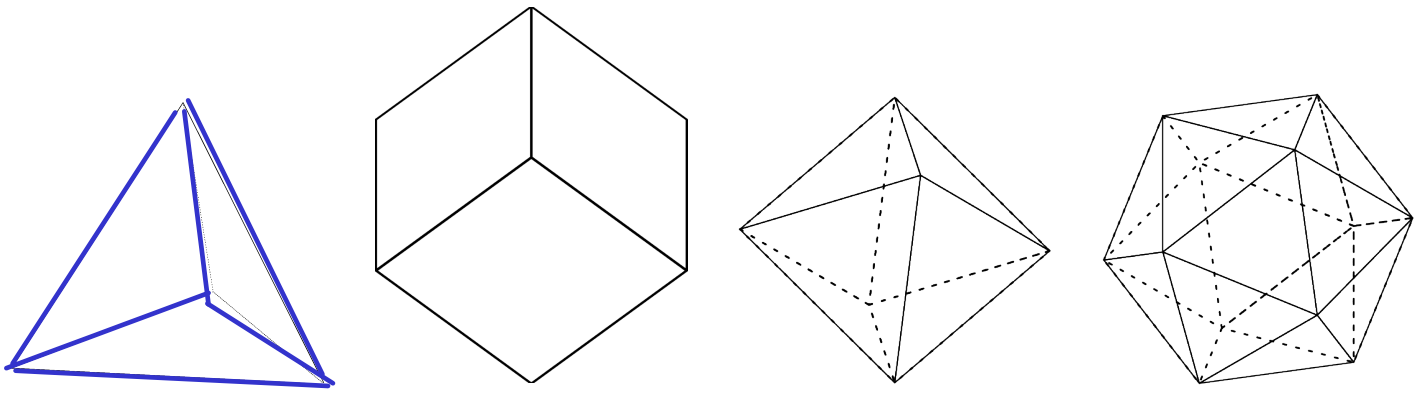
We'll solve problems like the two above, and also:

1. What are your chances of winning the Irish Lottery ("Lotto"). That is, what is the probability of correctly selecting **6** numbers from **47**?
2. If 500,000 people play the Lotto per week. What is the chance of a roll-over (i.e., nobody winning)?
3. For last night's men's international soccer match between Ireland and Serbia, a **23**-man squad was named.
   How many different ways were there of selecting the 11 starting players for the match?
   How many ways could one select (up to 5) of these players to be substituted during the game?
4. My password has 10 characters. Each character is an upper- or lower-case letter, or a digit. How long would it take you to crack my account?

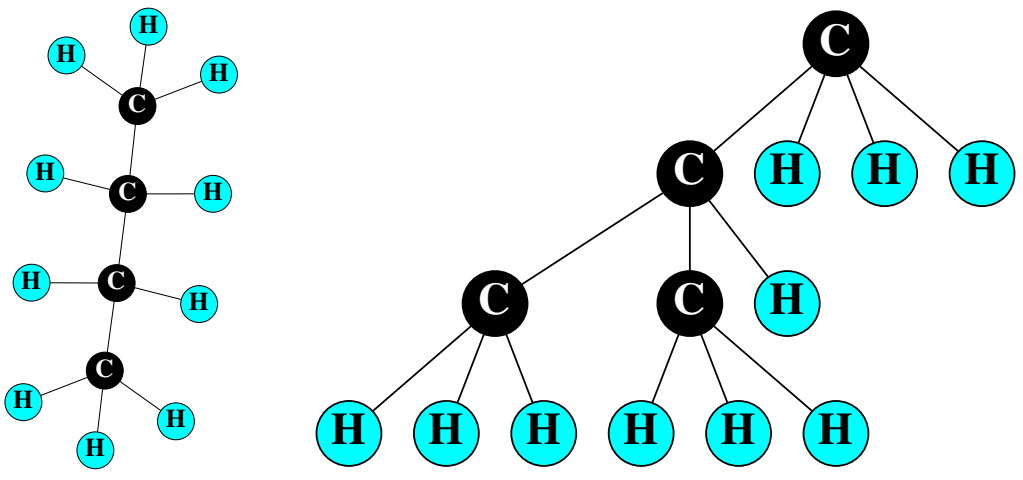


1. Which of these graphs are the **same** (and what does that mean)?
2. Is it possible to draw all the graph on the left so that none of its edges intersect? (Planar).
3. What is the smallest number of colours needed to colour every vertices so that no two adjacent vertices have the same colour?
4. Is there a "route" through the graph that visits every vertex once and only once?

5. How may regular polyhedra (platonic solids) are there?



6. Are all the graphs of *saturated hydrocarbon isomers* trees?



The most important reason for taking this module is that **Discrete mathematics is one of the most appealing, elegant, and applicable areas of mathematics**.

Appealing: The problems that we will consider are, I believe, easily motivated, but not trivial.

Elegant: The solutions to these problems involve some clever reasoning, but never tedious calculations.

Applicable: In spite of its classical origins, graph theory is one of the hottest topics in both pure and applied mathematics, with applications to network science, computer science, linguistics, chemistry, physics, biology, social science, and music.

Finished here Wed at 1.50.

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END OF PART 2