Topology: Beyond a First Syllabus

Each MA342 student is required to contribute a co-authored article which will be published online as a book with title *Topology: Beyond a First Syllabus*. This contribution counts for 15% of the module assessment.

Submission details

1. Each article should be no more than five pages maximum (including bibliography), should be written on a topology topic not covered in the MA342 exam syllabus, and should be written with a view to helping other MA342 students broaden their understanding of topology. The intended readership is MA342 students, and not professors of topology!

2. Each article should have at least two and no more than three co-authors. Let me know if you'd like help finding a co-author.

   All co-authors will receive a common score for their contribution.

3. The article should be submitted as a pdf file produced using the Latex style file for the Proceedings of the American Mathematical Society. Please do not adjust the default page size and font size. The syle file is available in Overleaf.

   The pdf file should be named `MA342_Surname1_Surname2_Surname3.pdf` and emailed to `graham.ellis@nuigalway.ie` by **5pm on Tuesday 31 March, 2020**.

4. You must follow all the AMS guidelines, and in particular:

   (a) Choose the most appropriate communicating AMS editor.

   (b) Include an abstract.

   (c) Include a primary Mathematical Subject Classification, and a secondary classification if appropriate.

   (d) Include a bibliography, with each bibliographic entry being cited at least once in the body of the article.

Possible topics

The MA342 exam syllabus is defined by the problem sheet, the lectures, and recent past exams. You are free to write on anything that complements, or expands on, this syllabus and that will help to improve other MA342 students' general understanding of the area of topology. You could choose an interesting definition and illustrate it and its use. You could choose a theorem and illustrate what it says. You could state a theorem and present a proof. You could write about some aspect of the life of a topologist. You could write about a recent trend in topology. The possibilities are endless. A few specific ideas are listed below.

- Discuss the quote:

  > Topology! The stratosphere of human thought! In the twenty-fourth century it might just possibly be of use to somebody, but for the present ...
  >
  > —— Solzhenitsyn, In the First Circle

- Explain/illustrate what the Poincaré Conjecture says, and write about the history of its proof. A statement of the conjecture, which is now a theorem, can be found on the Wikipedia topology page.

- State and prove any theorem from the later chapters of M.A. Armstrong's book *Basic Topology*.

- State and illustrate one of the topology theorems listed on the Wikipedia *Theorems in Topology* page.
- State and prove any theorem from Aisling McCluskey's book *Undergraduate Topology: A Working Textbook*
- What is a knot, what are the main goals of knot theory, and what were the origins of knot theory? (See for instance M.A. Armstrong's book *Basic Topology*.)
- Explain/illustrate what is meant by the homology of a simplicial complex. How does it relate to the Euler characteristic of a simplicial complex? (See for instance M.A. Armstrong's book *Basic Topology*.)
- Explain the Mapper clustering algorithm due to Singh, Mmoli, and Carlsson. Use a Google search to get started. If you are really ambitious you could even give an example using the R-package TDAmapper.
- Explain/illustrate what persistent homology is, and how it is used to understand point cloud data sets. Use a Google search to get started. If you are really ambitious you could even give an example using the R-package TDA.
- Write about topologists and the Fields medal. Which topologists have won the medal? Why did they win it? To get started you could look at the Wikipedia page on the Fields Medal.
- And so forth ...