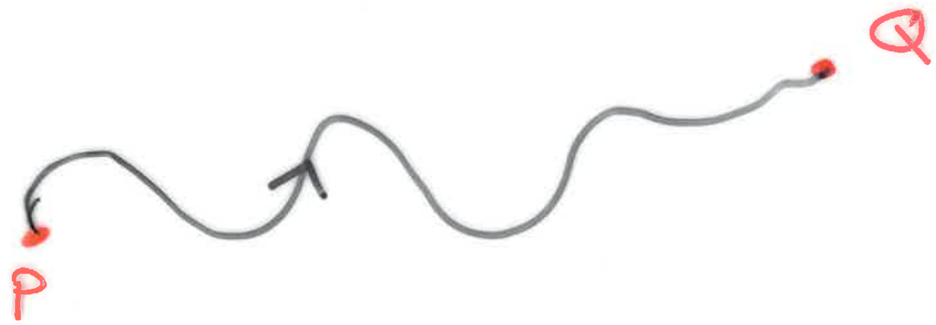
MA2286 Test: Wed 9 October

(up to and including Section 7)

The Fundamental Theorem of Calculus

Let ω be a differential 0-form on n-dimensional space.

Let S be a curve in $\mathbb{R}^n$ from P to Q.



Theorem

$$\int_S d\omega = \int_{\partial S} \omega$$

Example Evaluate

$$I = \int_S (y^3 + 2x)\,dx + 3xy^2\,dy$$

where S is the straight line from $P = (0,0)$ to $Q = (1,2)$.

Sol^n (using FTC)

Consider

$$\omega = xy^3 + x^2$$

Then

$$d\omega = (y^3 + 2x)\,dx + 3xy^2\,dy$$

by definition

So

$$I = \int_S d\omega = \int_{\partial S} \omega \overset{\downarrow}{=} \omega(Q) - \omega(P)$$

$$= 9 - 0 = 9.$$

Alternative Solution

The points $(x = t,\ y = 2t)$ traces out the straight line segment from $P = (0,0)$ to $Q = (1,2)$ as t goes from 0 to 1.

$x = t \qquad y = 2t$

$dx = dt \qquad dy = 2dt$

$$I = \int_0^1 ((2t)^3 + 2t)\,dt + 3t(2t)^2 2\,dt$$

$$= \int_0^1 32t^3 + 2t \quad dt$$

$$= \frac{32t^4}{4} + \frac{2t^2}{2} \Big|_0^1$$

$$= 9$$

<u>Problem</u> Evaluate

$$I = \int_S (6xy^2 - y^3)\,dx + (6x^2y - 3xy^2)\,dy$$

where S is some curve from $P = (1,1)$ to $Q = (3,4)$.

<u>Sol^n</u> Try to find $w = F(x,y)$ such that

$$dw = F_x\,dx + F_y\,dy$$

$$= (6xy^2 - y^3)\,dx + (6x^2y - 3xy^2)\,dy$$

$$F(x,y) = 3x^2y^2 - xy^3 + g(y)$$

$$F(x,y) = 3x^2y^2 - xy^3 + h(x)$$

we conclude that we need $g(y) = h(x)$ to be a constant c say.

$$I = \int_S d\omega$$

$$= \int_{\partial S} \omega$$

$$= \omega(Q) - \omega(P)$$

$$= F(3,4) - F(1,1)$$

$$= (3.9.16 - 3.4^3) - (3-1)$$

$$= 236.$$

Continuity

A function $f(x,y)$ is continuous if a small change in input yields only a small change in output.

More formally

We say that $f(x,y)$ is continuous at a point (x_0, y_0) if for any $\varepsilon > 0$ we find a $\delta > 0$ such that $f(x,y)$ is defined and

$$|f(x,y) - f(x_0,y_0)| < \varepsilon$$

whenever

$$|x - x_0| < \delta \text{ and } |y - y_0| < \delta$$