Example An investment portfolio involves two types of assets, type X and type Y.

It cost €3 to acquire one unit of asset X, and €-3 to relinquish one unit of asset X, it costs €4 to acquire one unit of asset Y, and -€4 to relinquish one unit of asset Y.

We say that the marginal costs are represented by the 1-form

$$\omega = 3dx + 4dy$$

**Example** Find the 1-form

$$\omega = A\,dx + B\,dy + C\,dz$$

describing work in the constant force field, where displacement of a particle from

$(0,0,0)$ to $(4,0,0)$ needs 3 units of work

$(1,-1,0)$ to $(1,1,0)$ needs 2 " " "

$(0,0,0)$ to $(3,0,2)$ needs 5 " " "

**Sol$^n$**

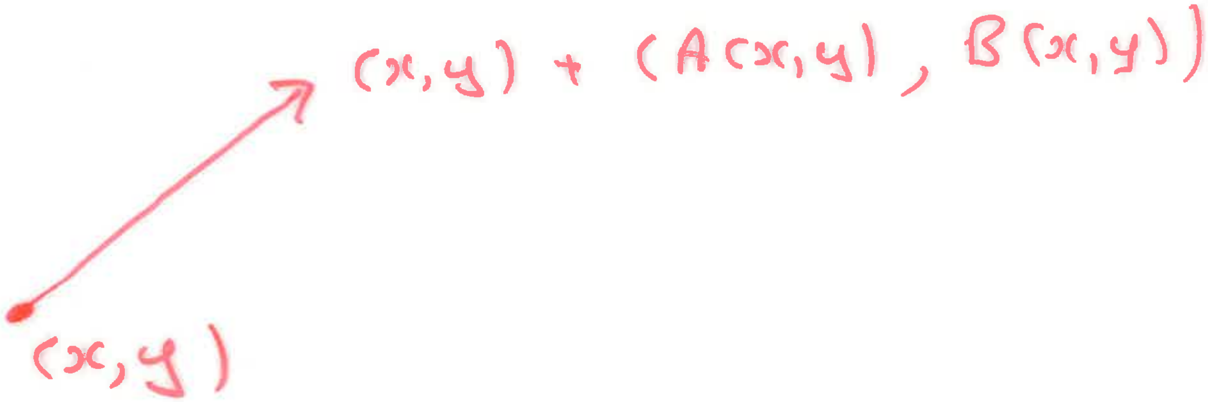
$$\left.\begin{aligned} 3 &= A.4 \\ 2 &= B.2 \\ 5 &= A.3 + C.2 \end{aligned}\right\} \qquad \begin{aligned} A &= \tfrac{3}{4} \\ B &= 1 \\ C &= \tfrac{11}{8} \end{aligned}$$

we can think of a 1-form

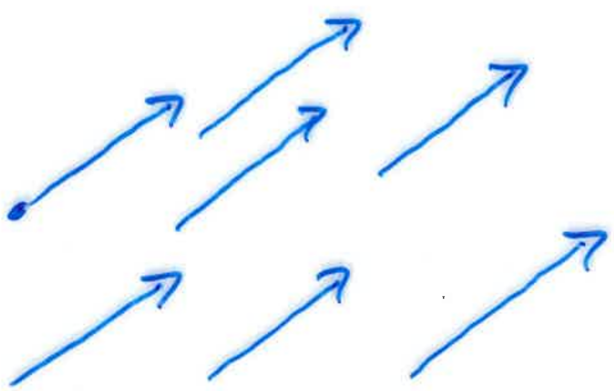
$$\omega = A(x,y)\,dx + B(x,y)\,dy$$

as a collection of arrows in space (= plane for two variables).

for each point $(x,y)$ in the space we have an arrow



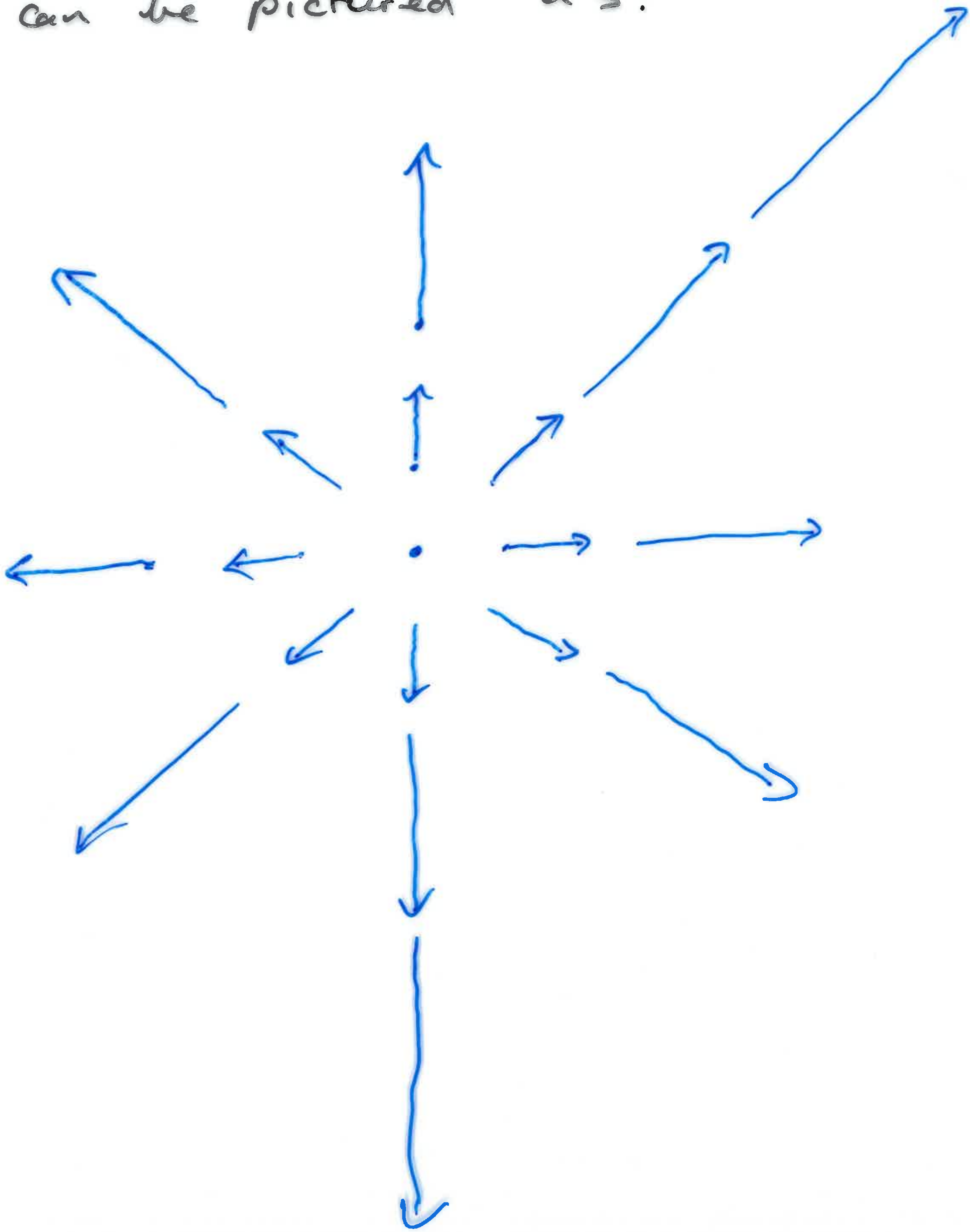

<u>Example</u> The 1-form $\omega = 2dx + dy$ can be pictured as



Example The 1-form

$$\omega = x\,dx + y\,dy$$

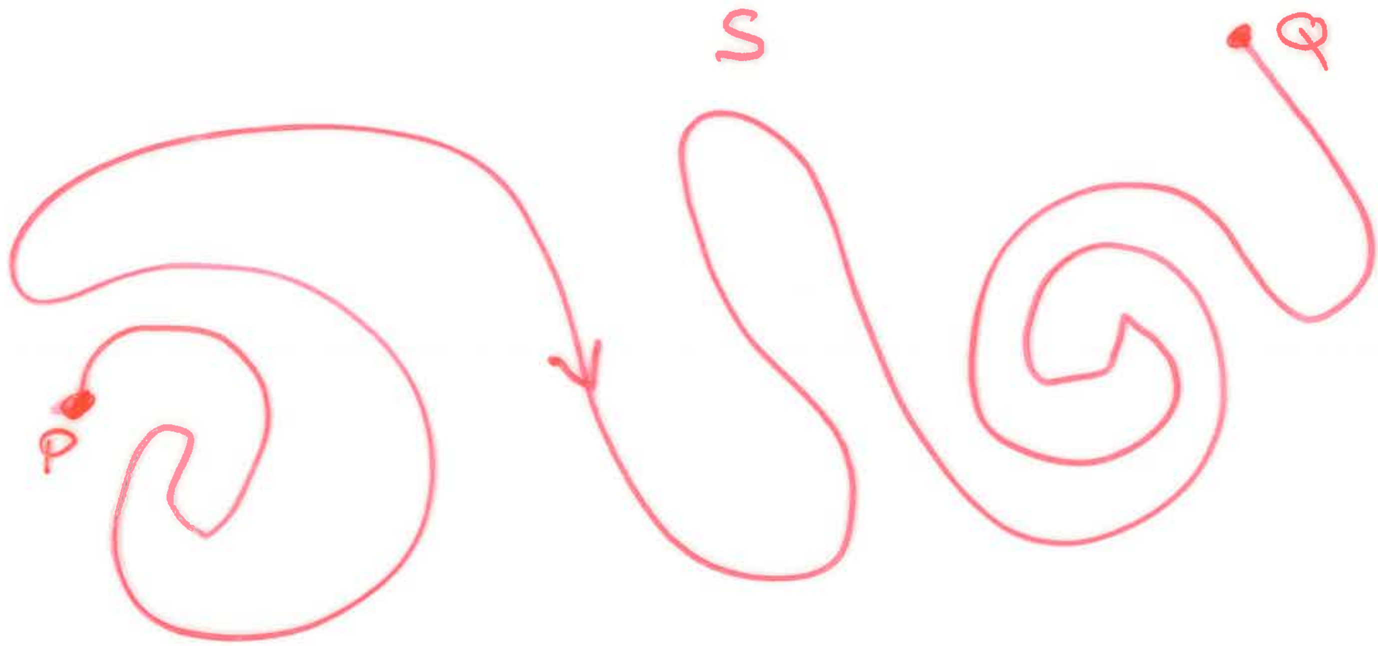
can be pictured as:



## Integration of 1-forms

Let $\omega = A(x,y)\,dx + B(x,y)\,dy$ be a differential 1-form.

Let $S \subseteq \mathbb{R}^2$ be a 1-dimensional, oriented, connected subset




<u>Informally</u>: If we think of $\omega$ as a work 1-form then

$$\int_S A(x,y)\,dx + B(x,y)\,dy$$

is the total work done in moving a particle from P to Q along S.