Definition / Example:

Integrate the 0-form $\omega = 3x^2 + 4$ on ∂S where

$S = [2,1] \cup [3,4]$.

Sol

$$\int_{\partial S} \omega = \int_{\partial[2,1]} \omega + \int_{\partial[3,4]} \omega$$

definition

$$= \omega(1) - \omega(2) + \omega(4) - \omega(3)$$

$$= 7 - 16 + 52 - 31$$

$$= 12.$$

Background Reading:

"Advanced calculus: a differential forms approach"

by Harold M. Edwards.

Also: Spivak's book on differential manifolds.

Stokes formula

$$\int_{\partial S} \omega = \int_{S} d\omega$$

$n=1$ variables

$p=0$, ω is a 0-form

Differential 1-forms in 1-variable ($p=1$, $n=1$)

A differential 1-form is a function of the form

$$\omega = f(x)\, h$$

which inputs two numbers $x, h \in \mathbb{R}$ and returns the number $f(x)\, h$, where $f(x)$ is some differentiable function.

Example Evaluate the 1-form

$$\omega = (x^2+6)\, h$$

at $x = 2$, $h = 0.5$

Sol^n

$$(2^2+6)\,0.5 = 5$$

Notation we usually denote the 1-form

$$\omega = f(x)\, h$$

by

$$\omega = f(x)\, dx$$

Example Evaluate the 1-form

$$\omega = \sin(x)\, dx$$

at $x = \frac{\pi}{2}$, $dx = 0.25$

Sol^n

$$\sin\left(\tfrac{\pi}{2}\right) 0.25 = 0.25$$

Defn Given a 1-form

$$\omega = f(x)\,dx$$

and an oriented interval $S=[a,b]$ we define the integral as

$$\int_S \omega = \underbrace{\int_a^b f(x)\,dx}_{\text{explained in 1st year}}$$

informally: $\int_a^b f(x)\,dx$ is the area between the curve $y=f(x)$ and the x-axis from a to b, where if $b>a$, areas above x-axis are regarded as positive and areas below x-axis are regarded as negative.

Problem A fundraising project has daily expenditure of \$ 10 000. The rate of contributions at time t is modelled by

$$c(t) = -100t^2 + 20\,000$$

what net proceeds can be expected.

Sol^n Project runs until

$$c(t) \leq 10\,000 .$$

$$-100t^2 + 20\,000 = 10\,000$$

$$100t^2 = 10\,000$$

$$t = 10 .$$

The project will run over the oriented interval

$$S = [0, 10] .$$

Contributions are modelled by the 1-form

$$\boxed{u = (-100\, t^2 + 20\,000)\, dt}$$

Expenditure is modelled by the 1-form

$$\boxed{v = -10\,000\, dt}$$

The net rate of income is modelled by the 1-form

$$\boxed{\omega = u + v = (-100\, t^2 + 10\,000)\, dt}$$

The project can be expected to make

$$\int_S \omega$$

where $S = [0, 10]$.

$$\int_S \omega \underset{\text{defn}}{=} \int_0^{10} (-100t^2 + 10000)\, dt$$

$$= -\frac{100t^3}{3} + 10000t \Big|_0^{10}$$

$$= \$66\,666.67$$