Differential 0-forms in 1-variable

$(p=0,\ n=1)$

A **differential 0-form** in 1 variable is just a differentiable real valued function

$$\omega = f(x)$$

Examples

$$\omega = 3x - 4$$

$$\omega = 3x^2+4$$

$$\omega = \sin(x)$$

are differential 0-forms.

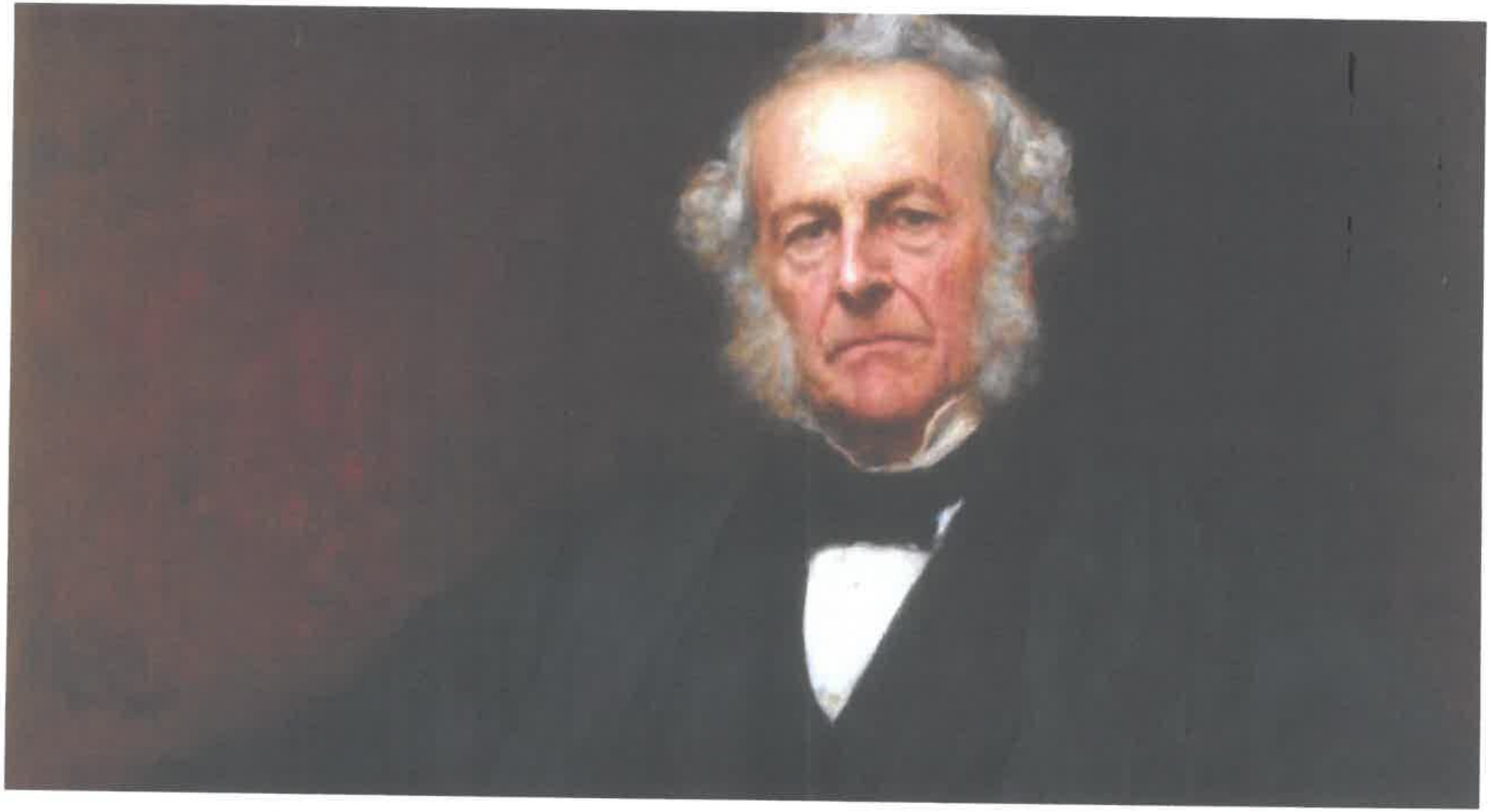
usually a differential 0-form is given in the context of some closed interval

$$S = [a,b] \subseteq \mathbb{R}$$

or a union of closed intervals

$$S = [a_1,b_1] \cup [a_2,b_2] \cup \dots \cup [a_k,b_k].$$

Sir George Stokes
1819 – 1903



William Thomson (Lord Kelvin)

1824 — 1907

9 Barton Street,
Westminster
July 2, 1850

My dear Stokes

As I have not a copy of your paper on the Equil. & Motion of Elastic Solids, nor any other work of reference for the purpose, by me, I shall be much obliged by your sending me the *equations of equil*^{m} of ⟨an⟩ a non crystalline elastic solid under the action of any forces, and the formulae for the mutual actions betw. any two contiguous portions of the body. I have been trying but as yet without success, to make out something about the integration of the equations for the case of a solid of any form, with each point of its surface displaced to a given extent & in a given direction from its natural position. I think I see how it can be done when the solid is a rectangular parallelepiped, but not in a very inviting way. It was reading your paper on diffraction on my way from Cambridge that made me take up the subject again.

Do you know that the condition that $\alpha dx + \beta dy + \gamma dz$ may be the diff^{l} of a function of two indep^{t} variables for all points of a surface is

$$l\left(\frac{d\beta}{dz}-\frac{d\gamma}{dy}\right)+m\left(\frac{d\gamma}{dx}-\frac{d\alpha}{dz}\right)+n\left(\frac{d\alpha}{dy}-\frac{d\beta}{dx}\right)=0?$$

I made this out some weeks ago with ref^{ce} to electromagnetism. With ref^{ce} to an elastic solid, the cond^{n} may be expressed thus – the resultant axis of rotation at any point of the surface must be perp^{r} to the normal.

Your's very truly
William Thomson

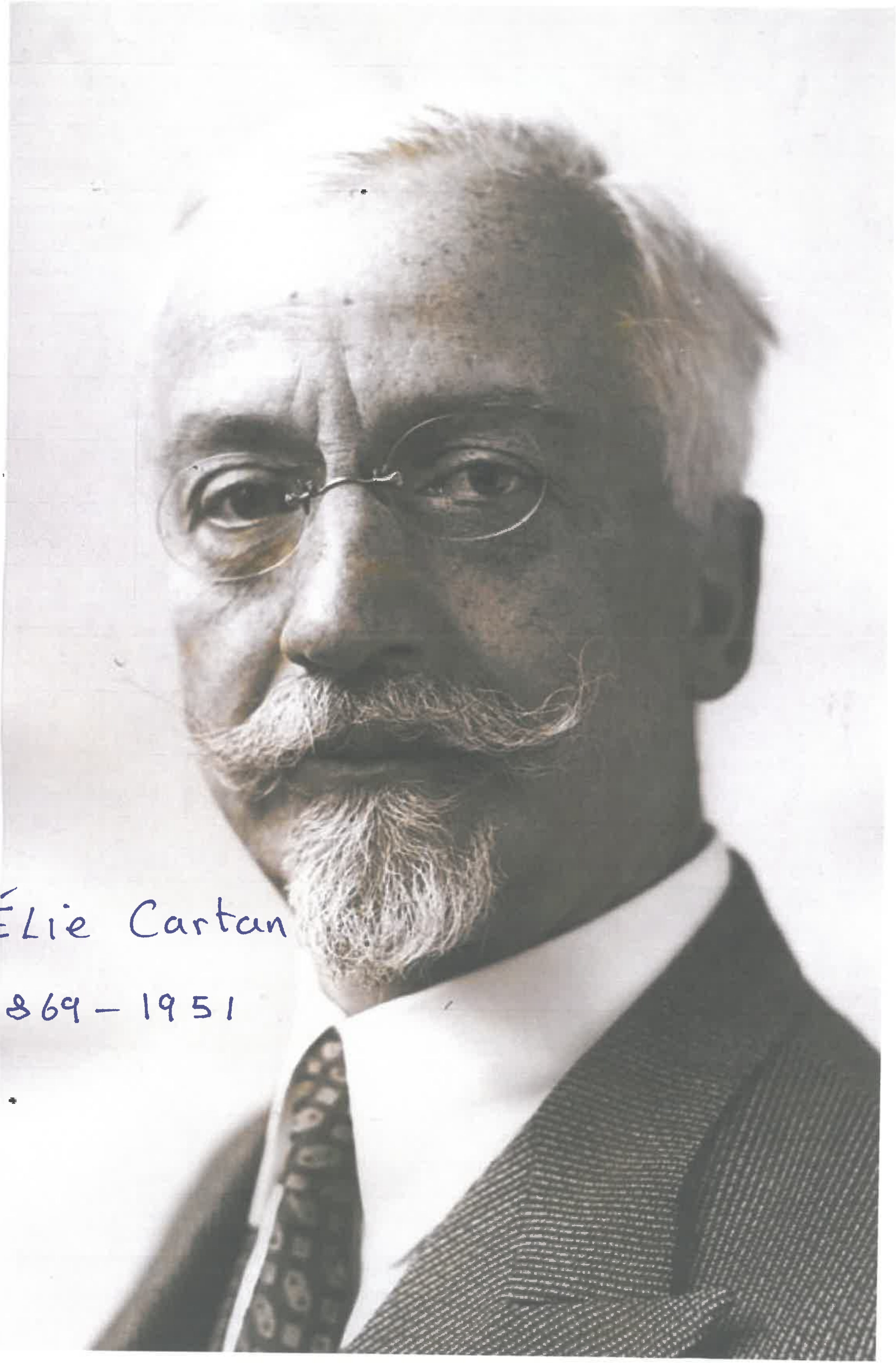
P.S. The following is also interesting, & is of importance with reference to both physical subjects.

$$\int(\alpha dx+\beta dy+\gamma dz)=\pm\iint\left\{l\left(\frac{d\beta}{dz}-\frac{d\gamma}{dy}\right)+m\left(\frac{d\gamma}{dx}-\frac{d\alpha}{dz}\right)+n\left(\frac{d\alpha}{dy}-\frac{d\beta}{dx}\right)\right\}dS$$

where l, m, n denote the dir^{n} cosines of a normal through any el^{t} dS of a ⟨limited⟩ surface; & the integ^{n} in the sec^{d} member is performed over a portion of this surface bounded by a curve round w^{h} the int^{n} in the 1st member is performed.

Sir William Rowan Hamilton

1805 – 1865



Élie Cartan

1869 – 1951

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$$\int_S d\omega = \int_{\partial S} \omega$$

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we only require ω to be differentiable on S.

Example

$$\omega = |x|$$

is a differential 0-form on

$$S = [1, 1000].$$

clearly ω is not a differential 0-form on

$$S = [-1, 1]$$

Terminology

I'll say 0-form instead of differential 0-form.

for $a < b \in \mathbb{R}$ we write

$$[a, b] = \{x \in \mathbb{R} : a \leq x \leq b\}$$

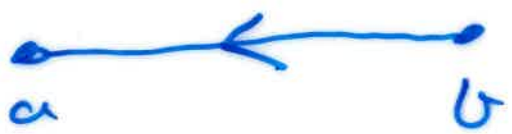
and we picture this as

The arrow is an orientation that specifies the direction of travel for a to b.

for $a < b \in \mathbb{R}$ we write

$$[b, a] = \{ x \in \mathbb{R} : a \leq x \leq b \}$$

and picture this as



We say that $[a, b]$ and $[b, a]$ are oriented intervals.

Example $S = [2,1] \cup [3,4] \cup [6,5]$

The boundary of the oriented interval $S = [a, b]$ is the set

$$\partial S = \{ a, b \}$$

the set consisting of two points, the initial point a and final point b.

Example

$S = [2,1] \cup [3,4] \cup [6,5]$

$\partial S = \{1, 2, 3, 4, 5, 6\}$.

Terminology we'll say that $S = [a, b]$ is 1-dimensional, and that the boundary is 0-dimensional.

Definition Given a 0-form

$$\omega = F(x)$$

on an oriented interval

$$S = [a, b]$$

we define

$$\int_{\partial S} \omega = F(b) - F(a).$$

Example Integrate the differential 0-form

$$\omega = 3x^2 + 4$$

over the boundary of the oriented interval

$$S = [2, 1].$$

Soln

$$\int_{\partial S} \omega = \omega(1) - \omega(2)$$

$$= 3(1^2)+4 - (3(2^2)+4)$$

$$= -9.$$