



Semester I Examinations 2017/2018

Exam Codes	1BA1, 1BCS1, 1BHR1, 1UPA1, 1BCT1
Exam Module	First Arts Examination, First Engineering Examination Analysis and Algebra 1
Module Code	MA133-1, MA160-1
Paper No.	1
External Examiner	Prof. Thomas Brady
Internal Examiners	Prof. Graham Ellis Dr. John Burns Dr. James Cruickshank
<u>Instructions</u>	Answer all six questions.
Duration	2 hours
No. of Pages	4 pages including this page
School	Mathematics, Statistics & Applied Mathematics
<u>Requirements</u>	No special requirements
Release to Library:	Yes
Other Materials	Non-programmable calculators

Q1. (a) Let $L: \mathbb{R}^2 \rightarrow \mathbb{R}^2$ be the linear transformation with matrix

$$A = \begin{pmatrix} -3 & 2 \\ 4 & 1 \end{pmatrix}$$

with respect to the standard basis vectors.

- (i) [4 marks] Find $L((5, 0))$ and $L((0, 2))$.
- (ii) [4 marks] Find the image of the line $2x + 5y = 10$ under the linear transformation L .

(b) Consider the matrix

$$A = \begin{pmatrix} 6 & 5 \\ -3 & -2 \end{pmatrix}.$$

- (i) [5 marks] Verify that $\begin{pmatrix} -1 \\ 1 \end{pmatrix}$, $\begin{pmatrix} -5 \\ 3 \end{pmatrix}$ are eigenvectors of A .
Hence find a diagonal matrix D and matrix E such that $A^n = ED^nE^{-1}$.
- (ii) [4 marks] Calculate A^8 .

Q2. (a) [8 marks] Encipher the plaintext

GAGA

by applying the enciphering function

$$f_E: \mathbb{Z}_{26} \rightarrow \mathbb{Z}_{26}, x \rightarrow 14x + 12$$

to 1-letter message units over the alphabet $A = 0, B = 1, \dots, Z = 25$. Why is there a problem finding the corresponding deciphering function?

(b) [9 marks] Find the determinant and inverse of the matrix

$$A = \begin{pmatrix} 1 & 0 & 1 \\ 4 & 1 & 5 \\ 4 & 1 & 4 \end{pmatrix}.$$

Q3. (a) (i) [3 marks] Use Euclid's Algorithm to find $\gcd(7, 27)$.

(ii) [4 marks] Find 7^{-1} in \mathbb{Z}_{27} .

(b) [5 marks] Determine the third digit of the ISBN 3-5?0-90336-4.

(c) [5 marks]

Solve the simultaneous congruences

$$x \equiv 3 \pmod{7}$$

$$x \equiv 4 \pmod{9}.$$

Q4. (a) [12 marks] Evaluate the following limits.

(i) $\lim_{x \rightarrow +\infty} \frac{x+1}{2-x}$.

(ii) $\lim_{x \rightarrow -2} \frac{x^2 - 3x - 10}{x^2 + 3x + 2}$.

(iii) $\lim_{x \rightarrow 0} \frac{\sin^2 4x}{1 - \cos x}$.

(b) [5 marks] Sketch the graph of the function $f(x) = \frac{x+1}{2-x}$.

Q5. (a) [5 marks] Let $f(x) = \frac{1}{x}$. Show, using the definition of the derivative, that $f'(1) = -1$.

(b) [12 marks] Find the derivative of each of the following functions.

(i) $f(x) = 2x^3 - 5x^2 + x + 1$.

(ii) $g(x) = x^2 \sin x \cos x$.

(iii) $h(x) = e^{x^2}$.

Q6. (a) [9 marks] Find the critical points of the function

$$f(x) = 2x^3 + 9x^2 + 12x + 5$$

and use the second derivative test to classify each of them as either a local minimum point or a local maximum point.

(b) [8 marks] Find the maximum value of xy where x and y are real numbers satisfying

$$x^2 + \frac{y^2}{4} = 2.$$