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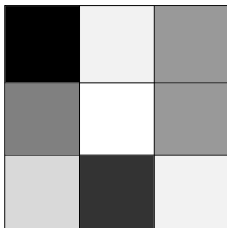
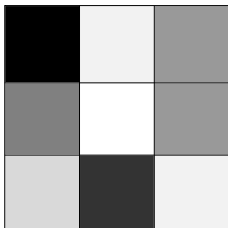


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By normalizing with respect to brightness and contrast the patches are projected onto a set \mathcal{S} of points in a topological seven-sphere $\mathbb{S}^7 \subset \mathbb{E}^8$.

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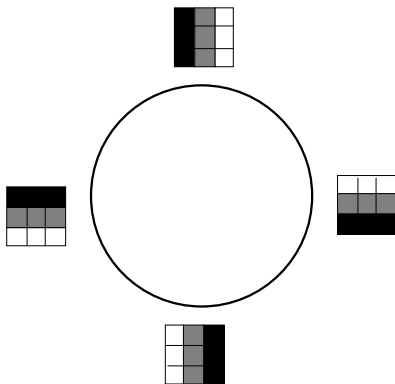
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$\mathcal{S}(k)$ contains the 25% of patches with least $\delta_k(x)$.

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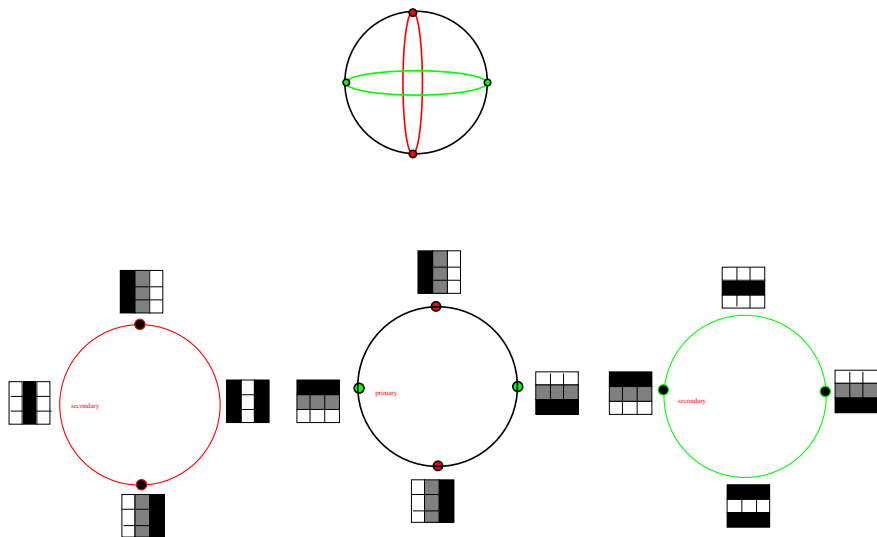
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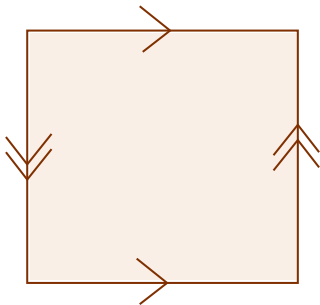
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